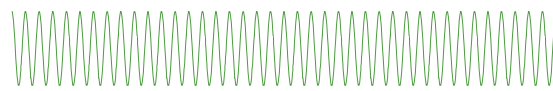


# Time-Frequency analysis of biophysical time series

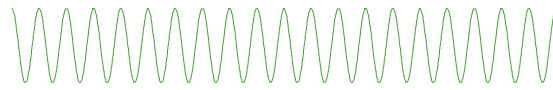
© Arnaud Delorme  
CERCO, CNRS, France & SCCN, UCSD, La Jolla, USA

# Frequency analysis

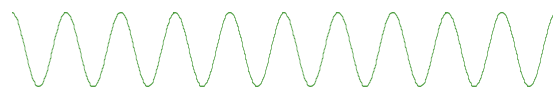
synchronicity of cell  
excitation determines  
amplitude and rhythm  
of the EEG signal



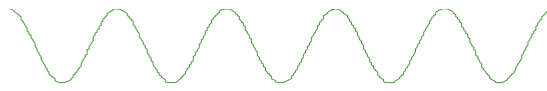
30-60 Hz Gamma



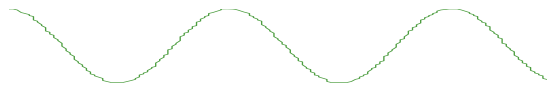
18-21 Hz Beta



9-11 Hz Alpha



4-7 Hz Theta

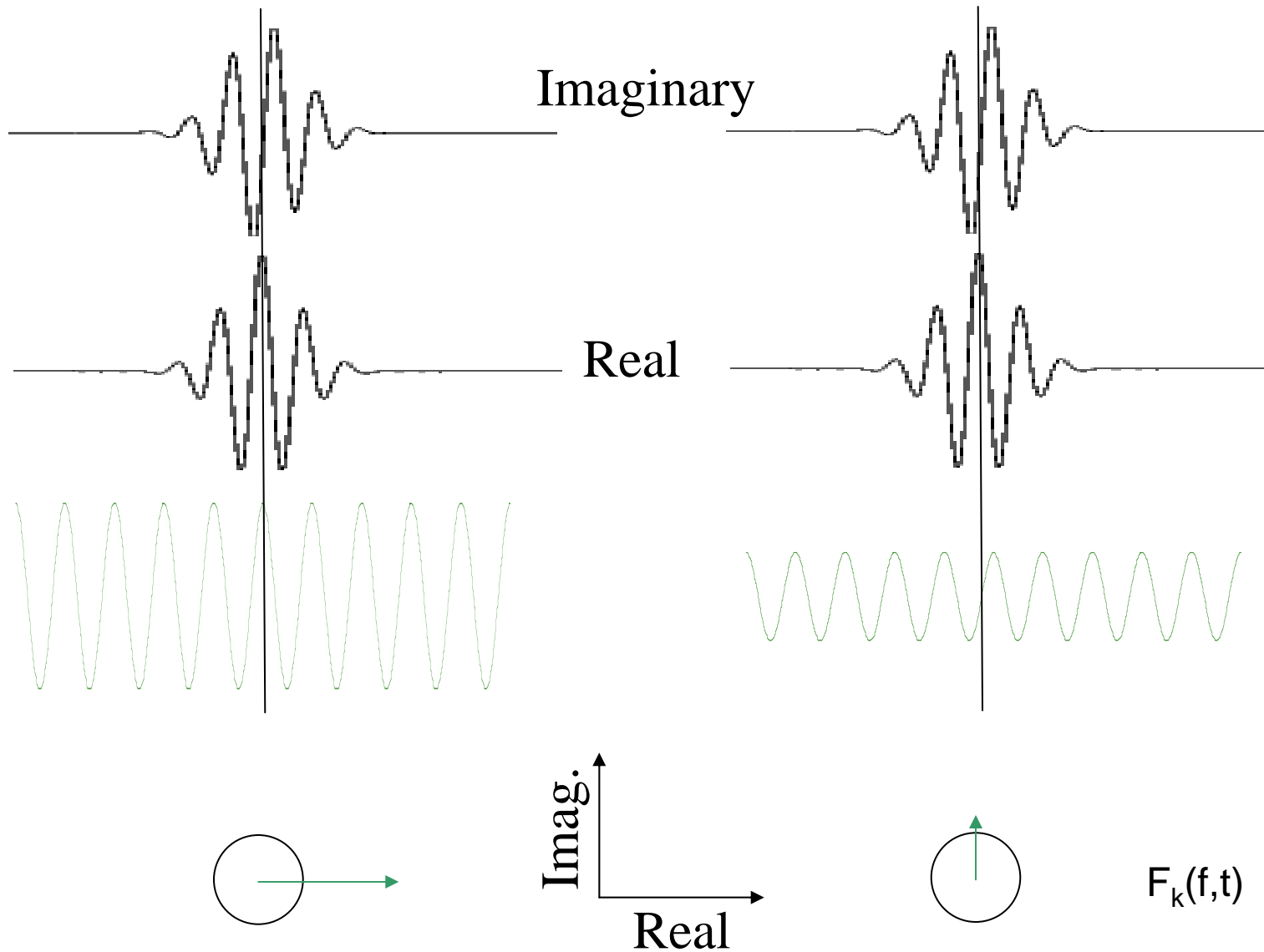


0.5-2 Hz Delta

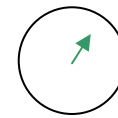
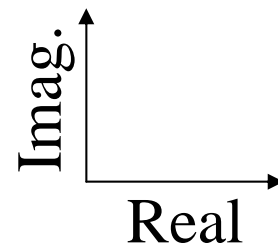
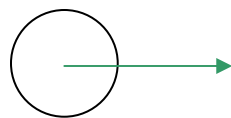
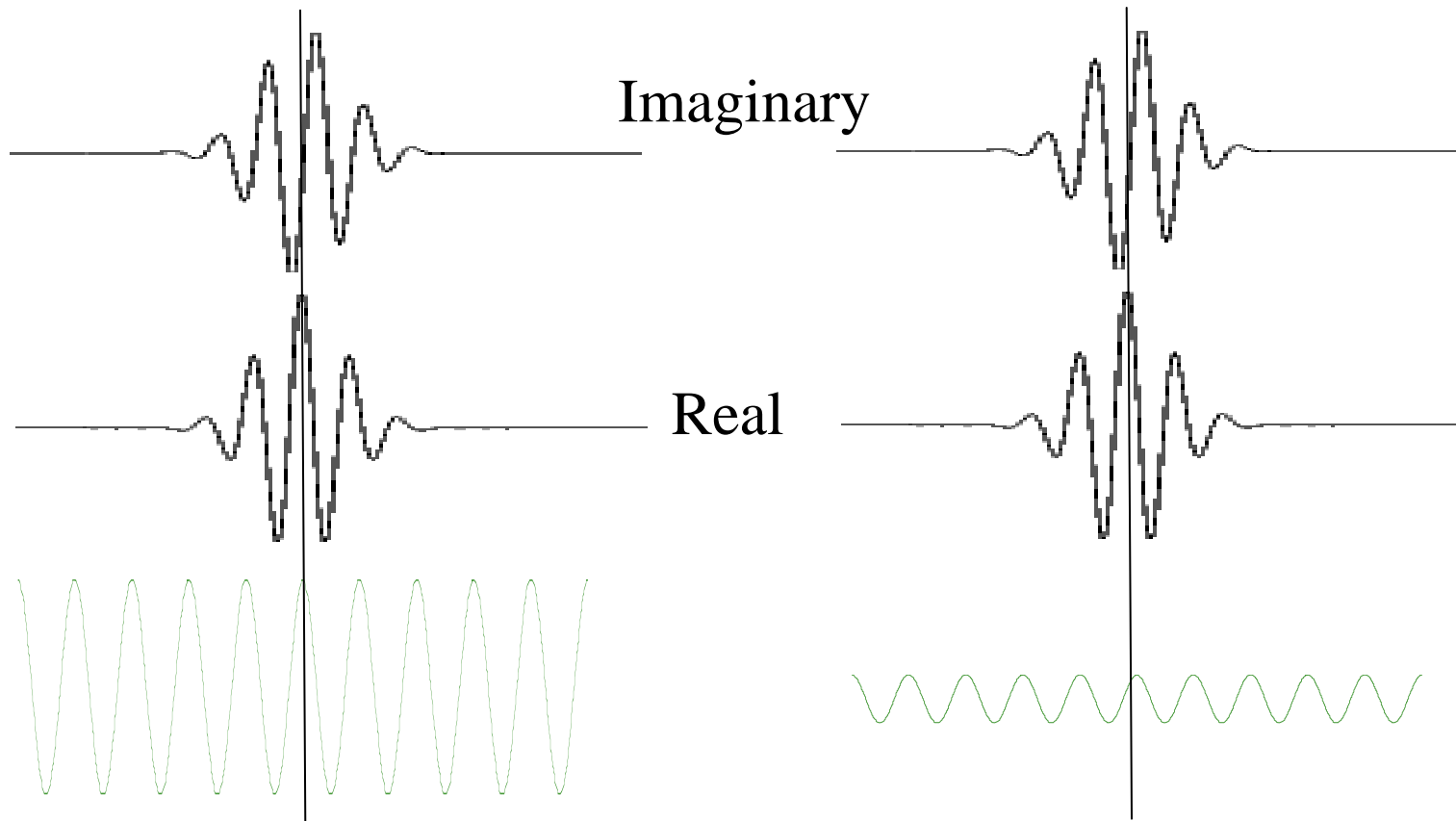


1 second

# Spectral phase and amplitude



# Spectral phase and amplitude



$F_k(f,t)$

## function [a,b] = dft (y)

```
%      DFT - The Discrete Fourier Transform
%      [a, b] = DFT (y)
%      a, b are the cosine and sine components
```

```
n = length (y);
t = 2*pi*(0:n-1)/n;
f = 2.0 / n;
```

```
for j = 0:n2
```

```
    cs = cos (j * t);
```

```
    ss = sin (j * t);
```

```
    a(j+1) = f * (cs * y);
```

```
    b(j+1) = f * (ss * y);
```

```
end
```

```
% boundaries
```

```
n2 = floor (n / 2);
```

```
a(1) = 0.5 * a(1);
```

```
a(n2+1) = 0.5 * a(n2+1);
```

```
b(1) = 0.0;
```

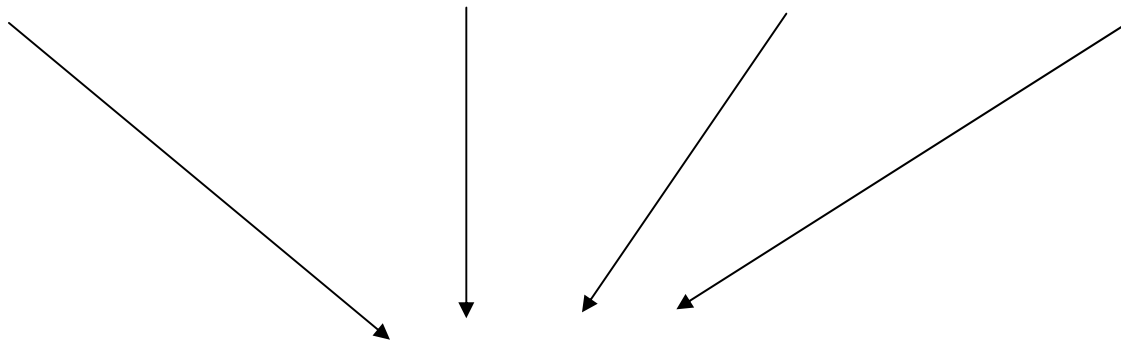
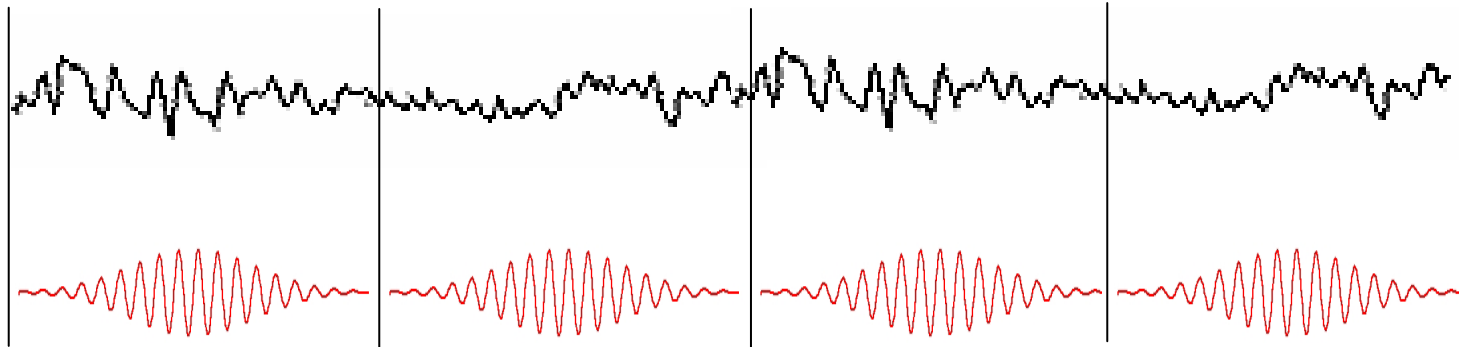
```
b(n2+1) = 0.0;
```

Loop on frequency

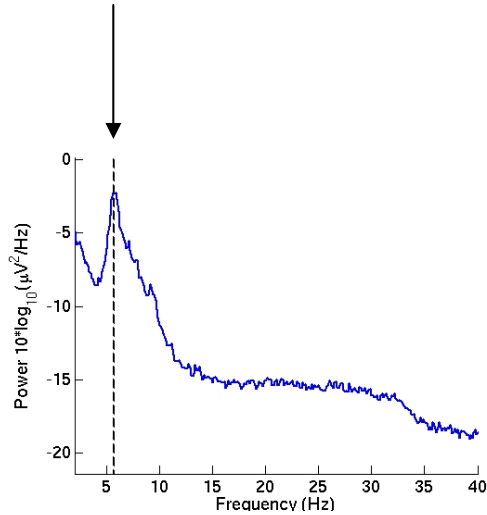
Cosine component

Sine component

Multiply with signal

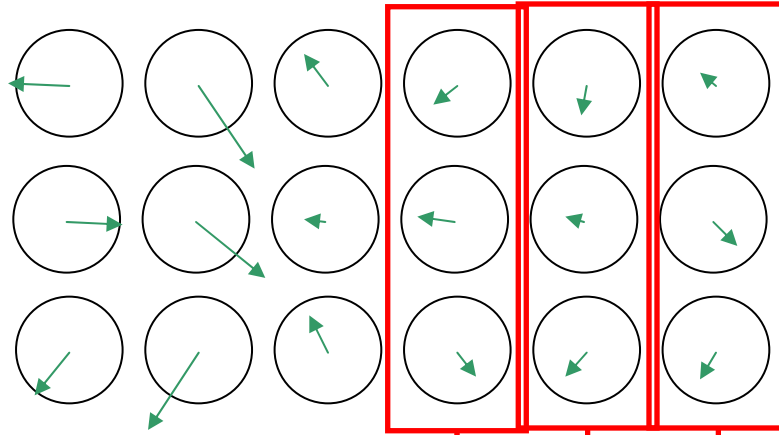


Average of squared absolute values

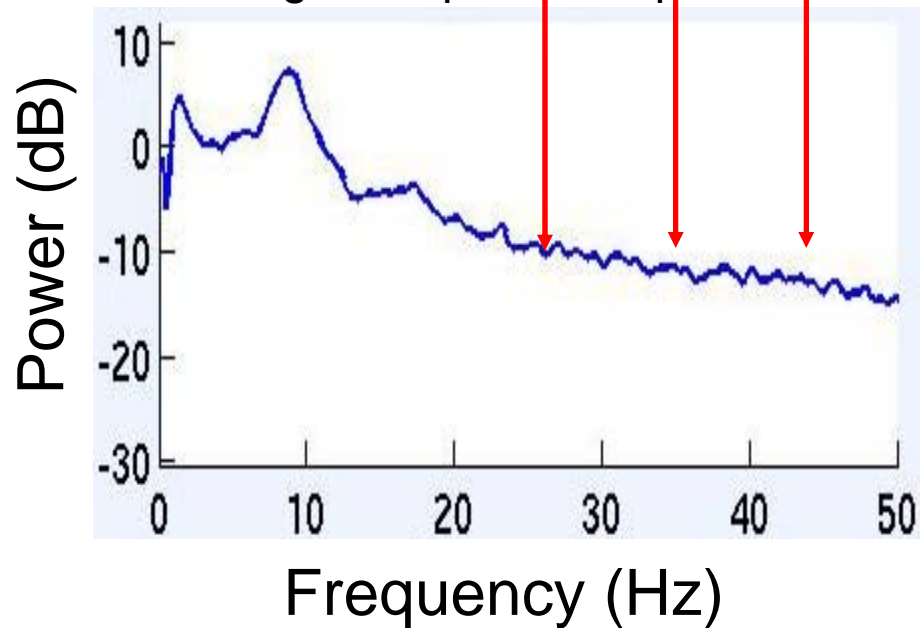


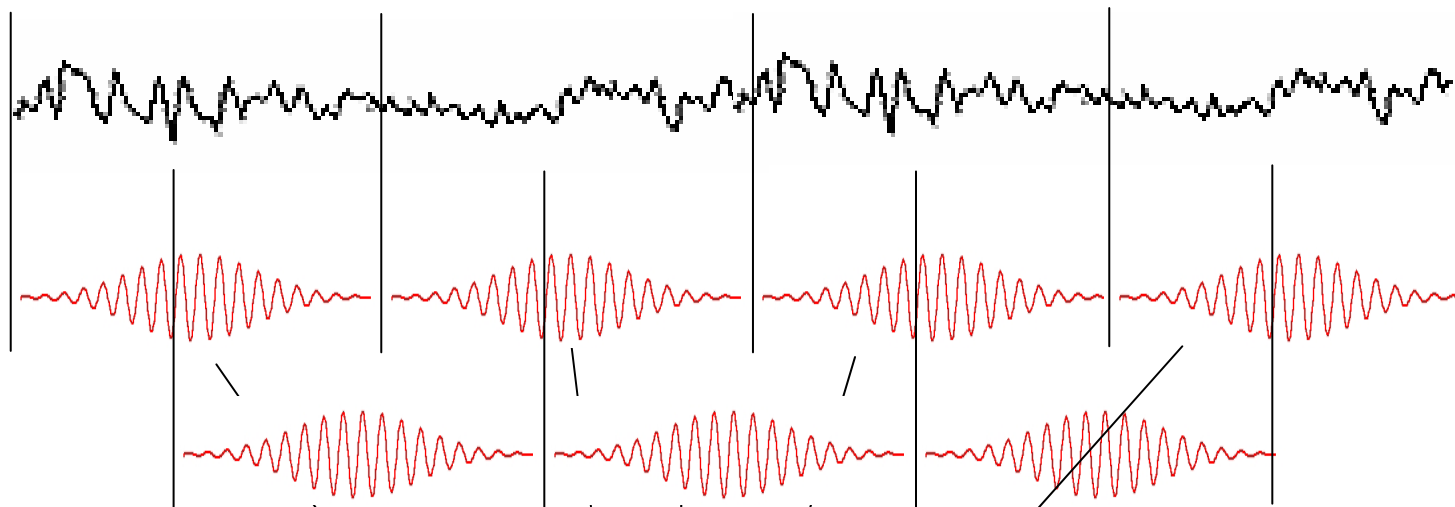
# Spectral power

0 Hz 10 Hz 20 Hz 30 Hz 40 Hz 50 Hz



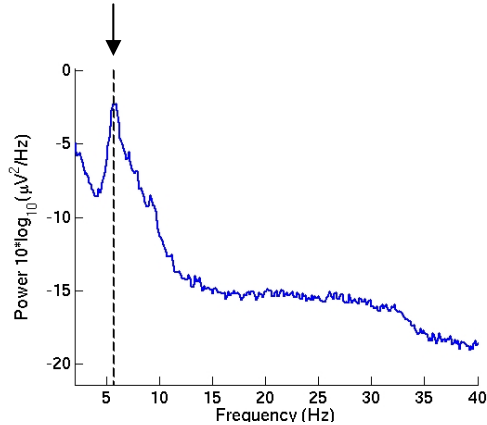
Average of squared amplitude

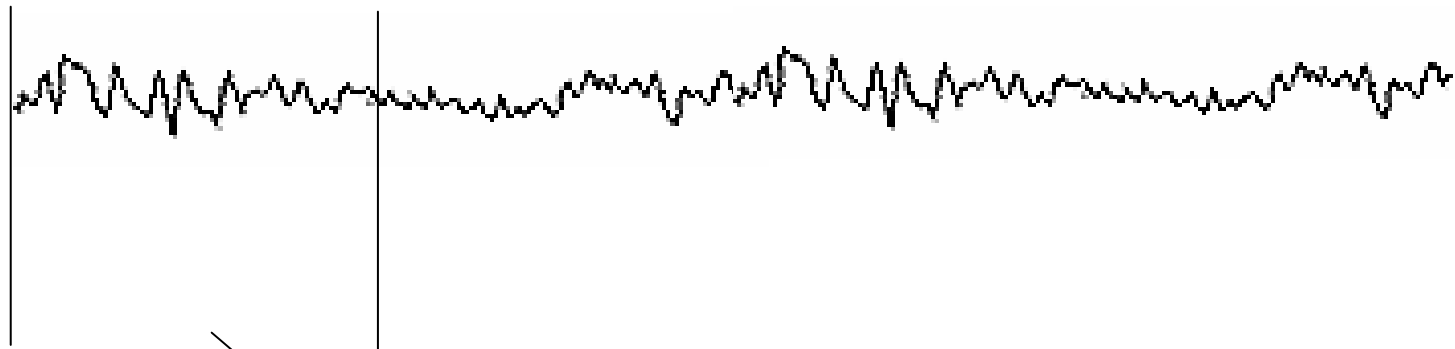




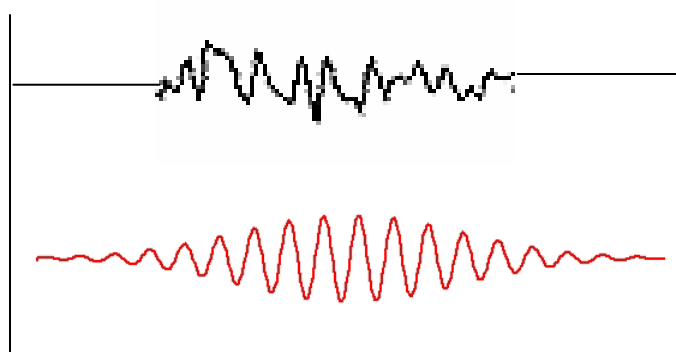
**Overlap 50%**

**Average**

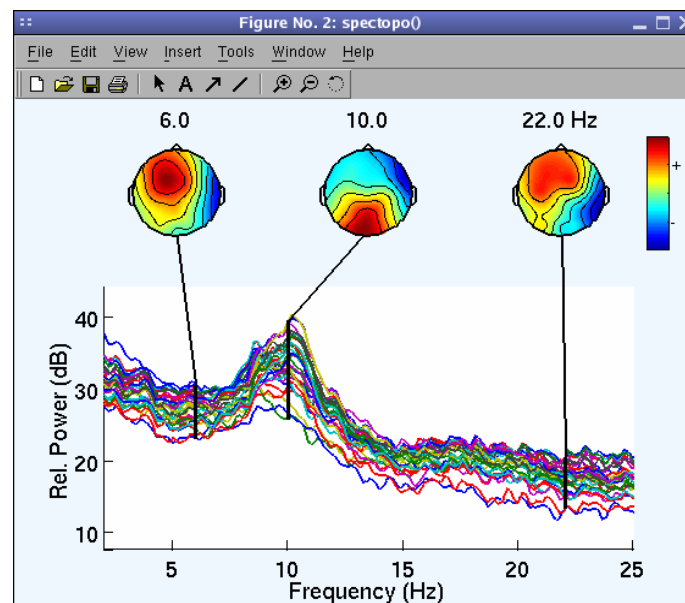
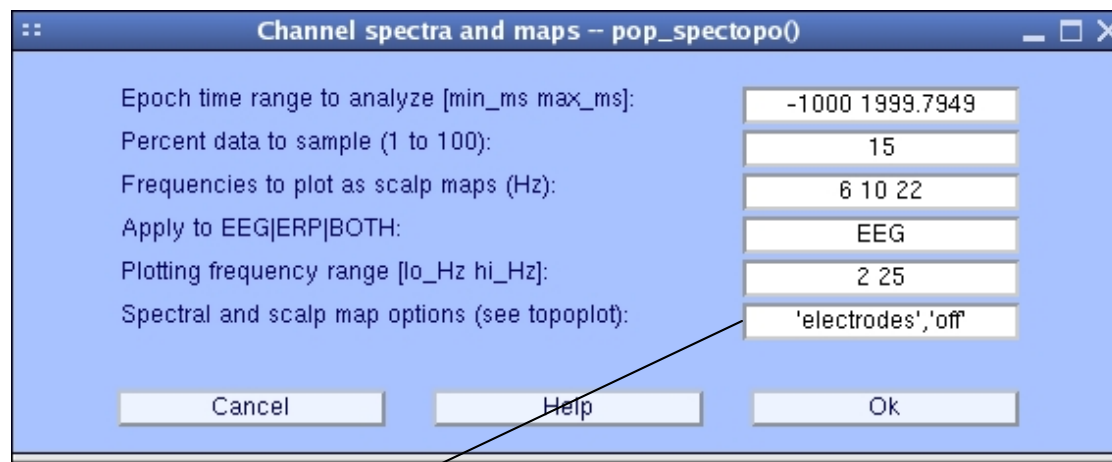
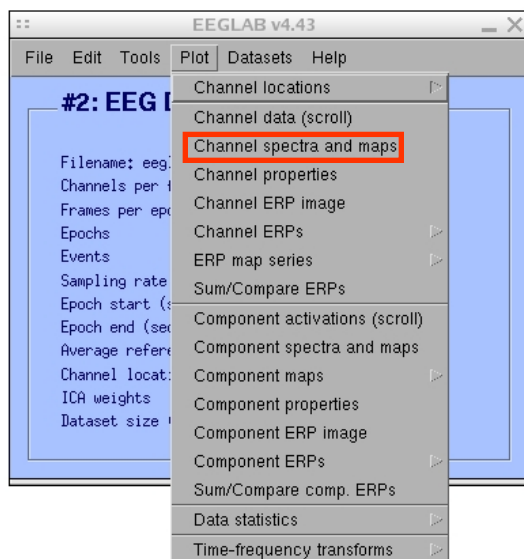




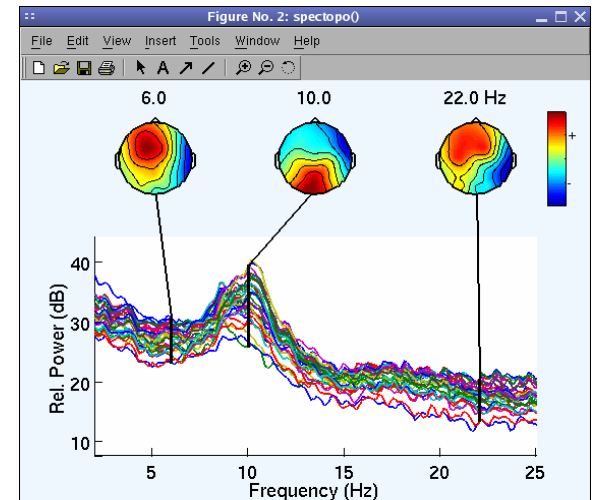
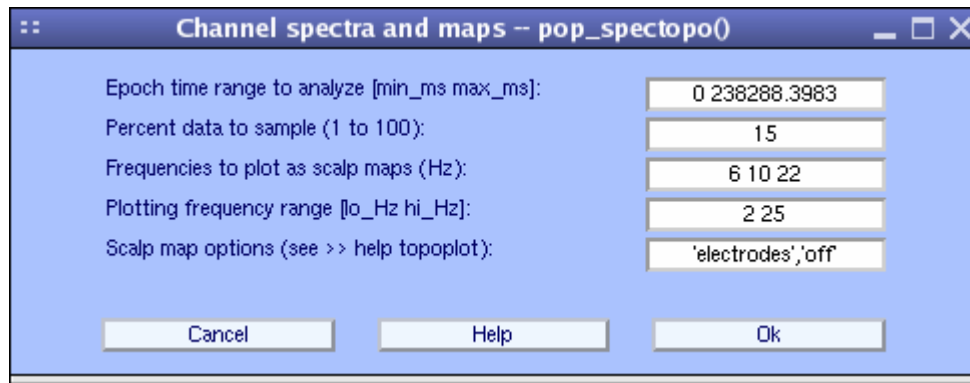
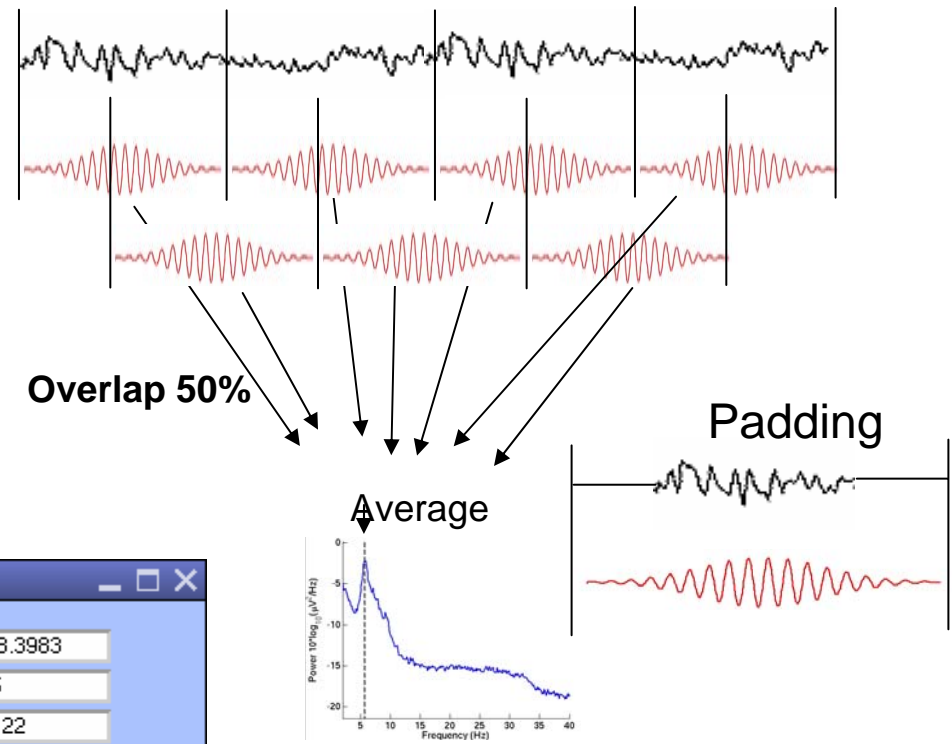
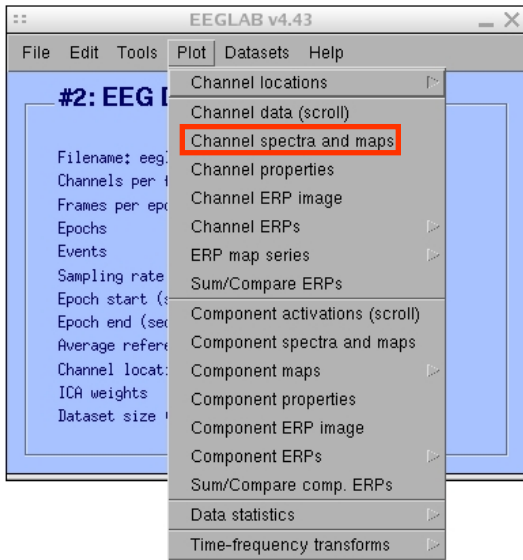
**padding**



# Plot data spectrum using EEGLAB



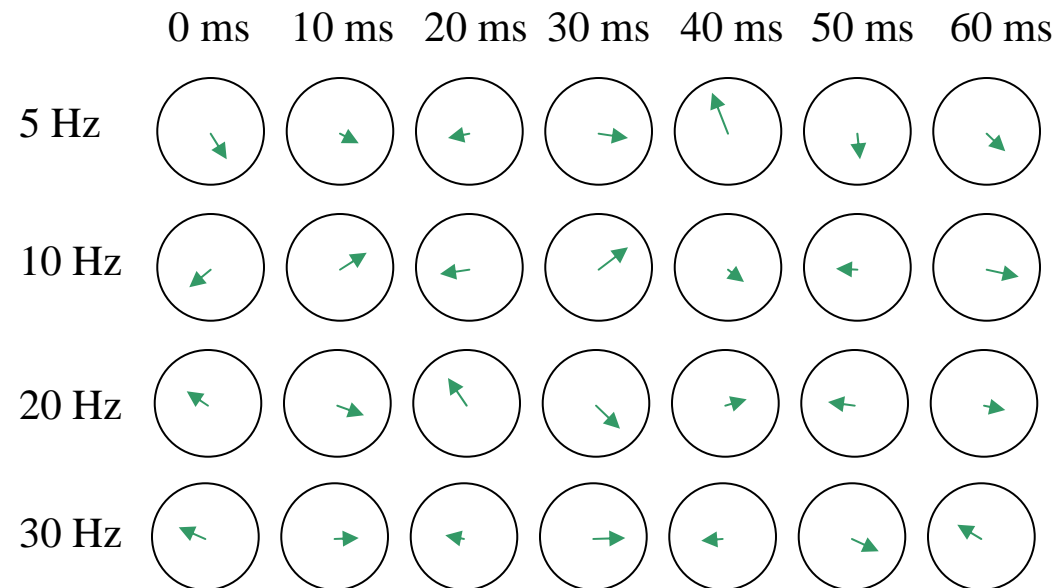
**'winsize', 256** (change FFT window length)  
**'nfft', 256** (change FFT padding)  
**'overlap', 128** (change window overlap)



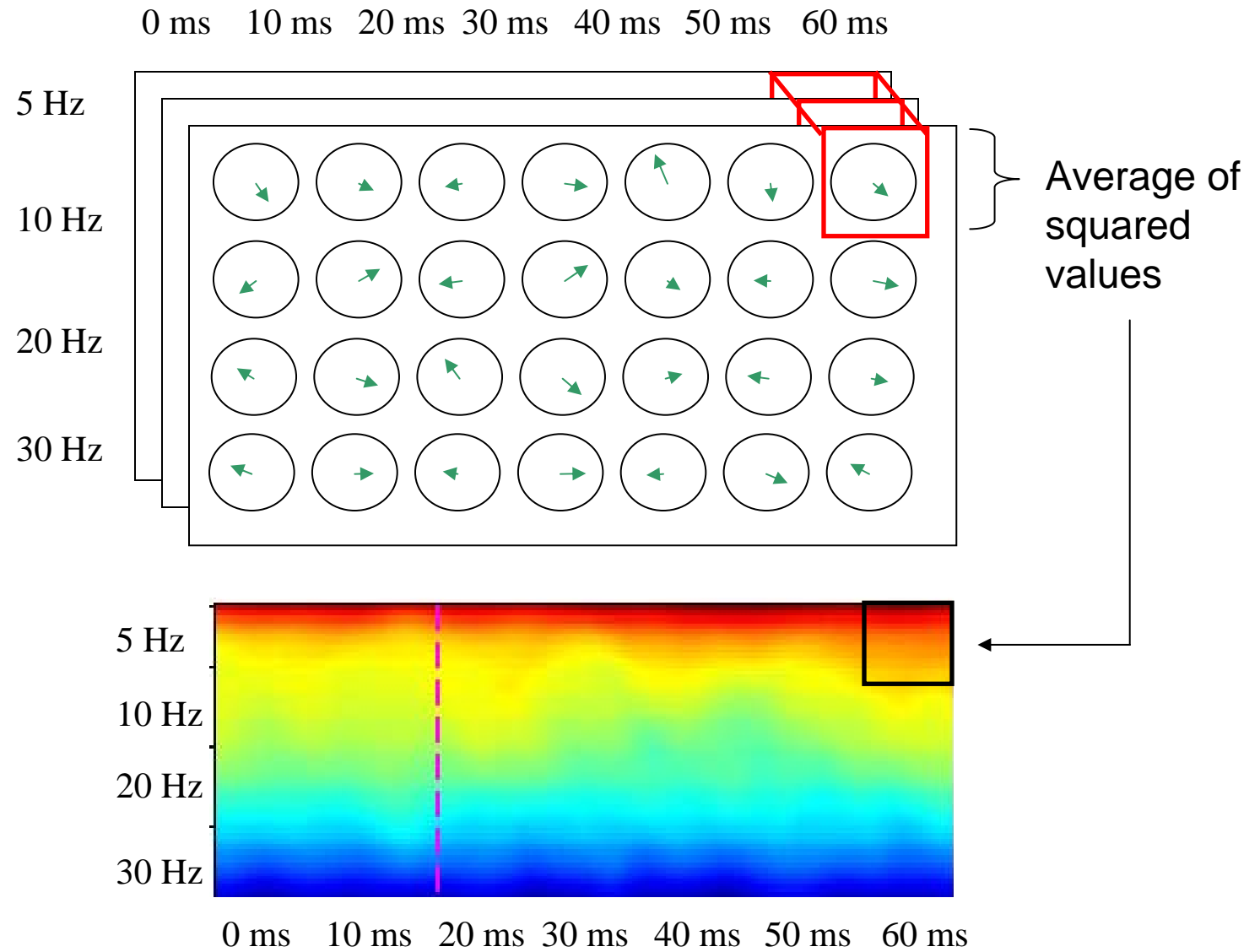
## Exercise

- Read dataset "eeglab\_data\_epoch\_ica.set"
- Plot spectral decomposition
  - 1- default (15% of data)
  - 2- with 100% of data
  - 3- with 100% and 50% overlap ('overlap')
  - 4- reduce window size ('winsize')
  - 5- reduce fft length ('nfft')

# Spectrogram or ERSP



# Spectrogram or ERSP



# Power spectrum and event-related spectral perturbation

$$ERS(f, t) = \frac{1}{n} \sum_{k=1}^n |F_k(f, t)|^2$$



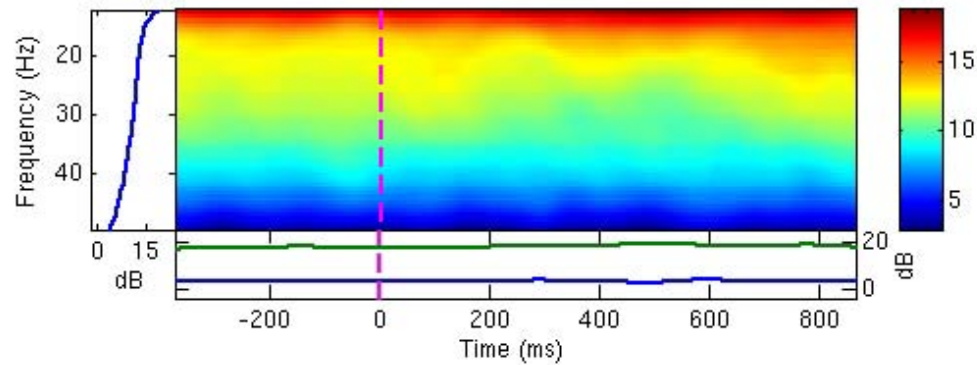
Scaled to dB  $10\text{Log}_{10}(\text{ERSP})$



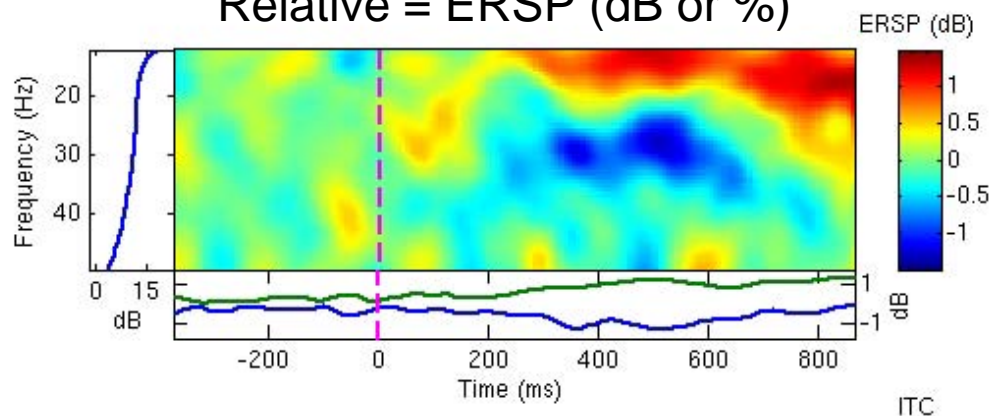
Complex number

# Absolute versus relative power

Absolute = ERS



Relative = ERSP (dB or %)

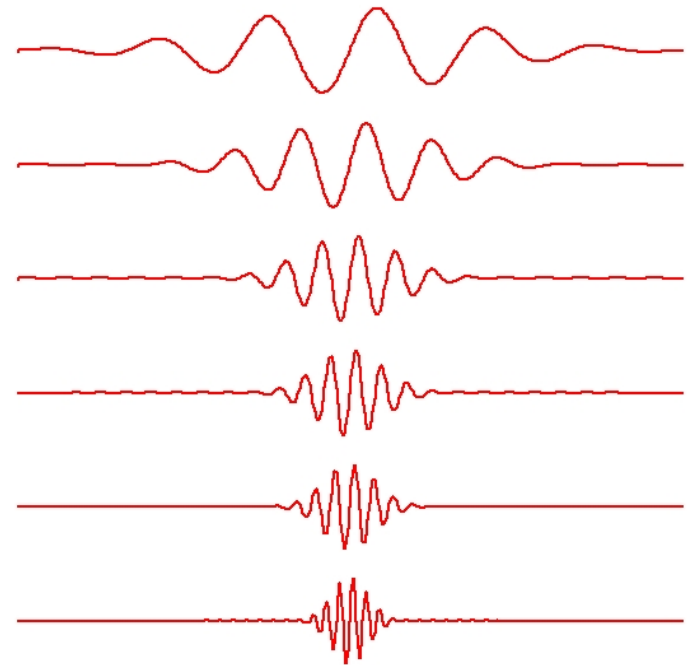
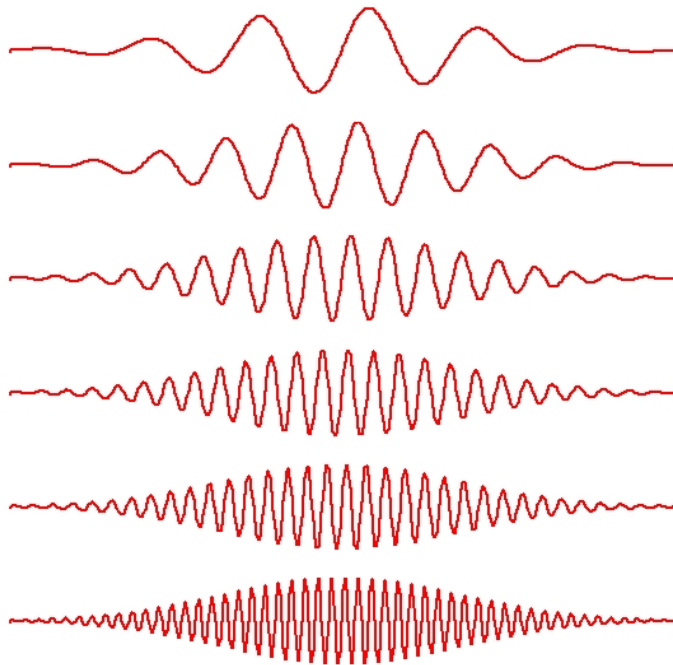


# Difference between FFT and wavelets

FFT

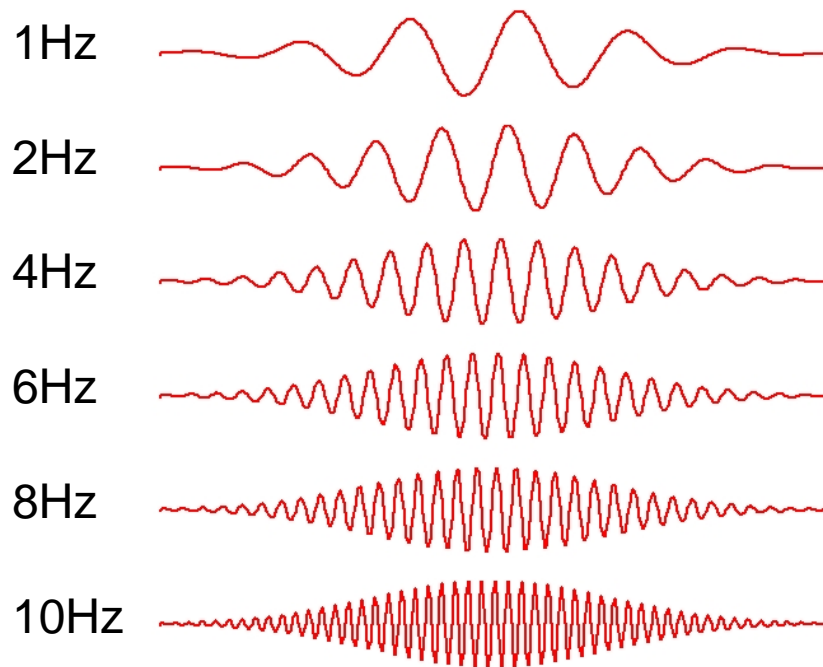
Wavelet

Frequency  
↓

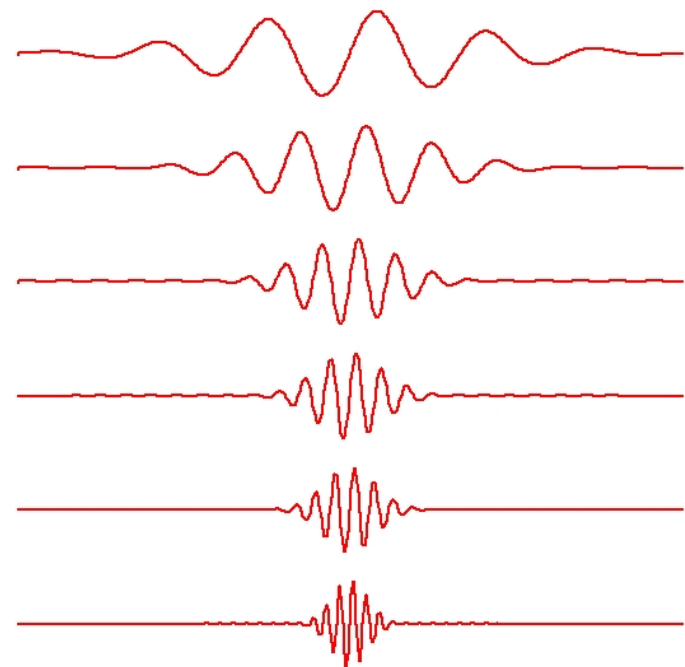


# Wavelets factor

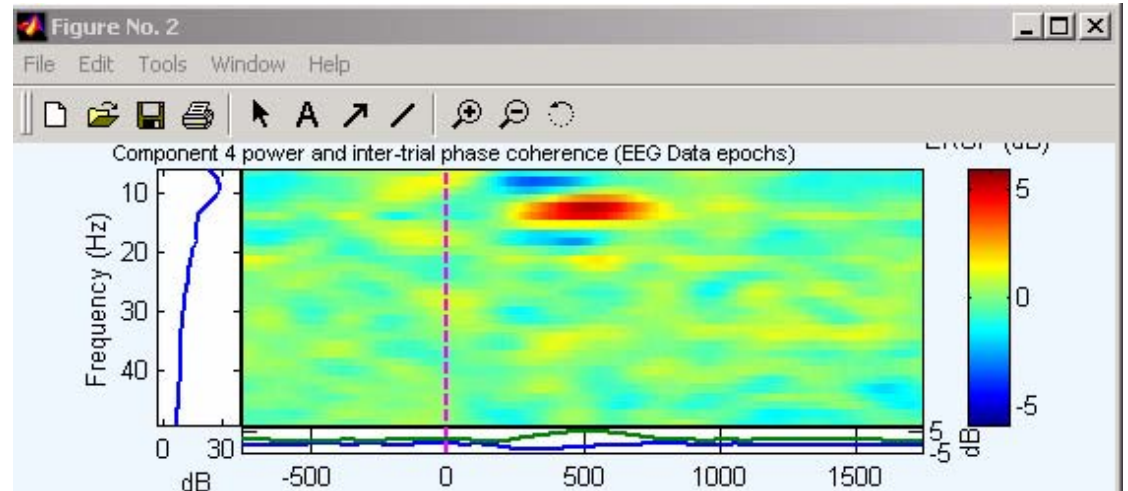
Wavelet (0)= FFT



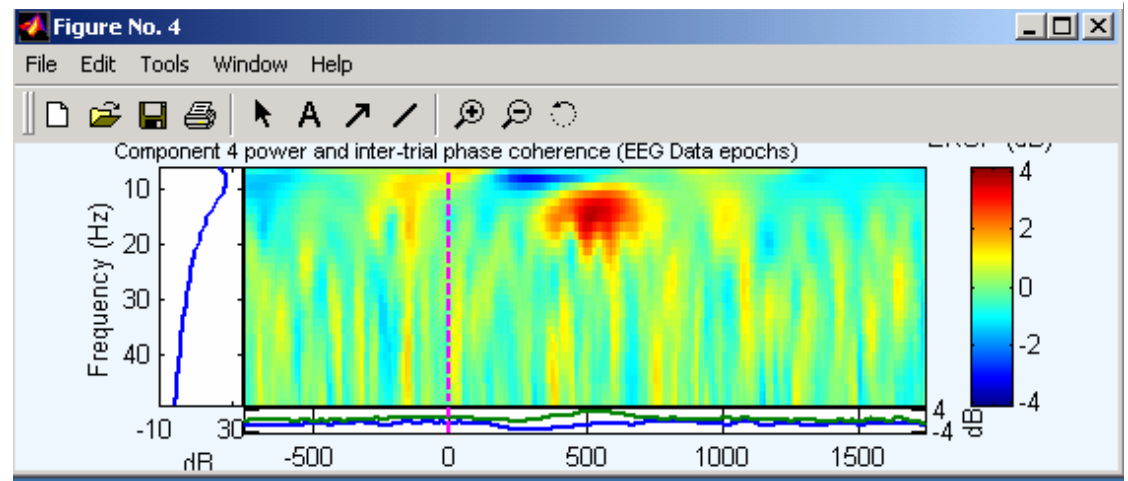
Wavelet (1)



FFT

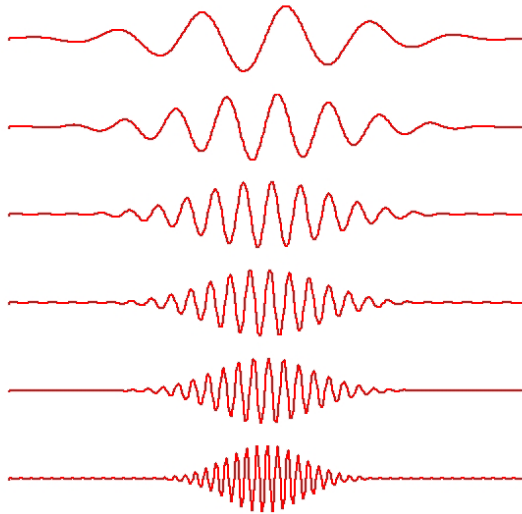


Pure wavelet

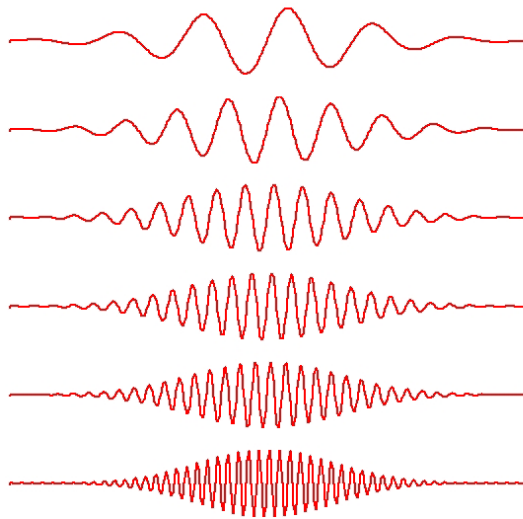


# Modified wavelets

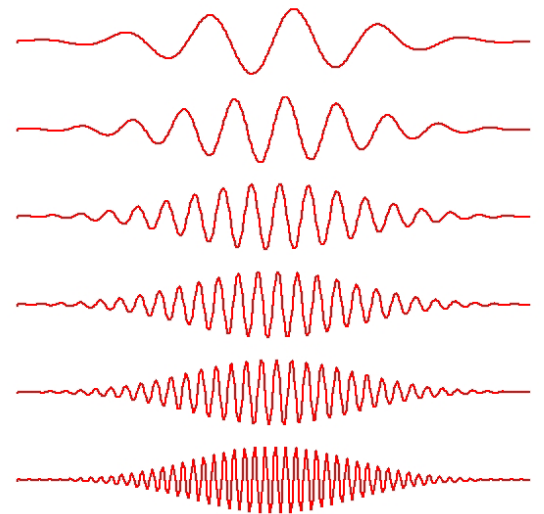
Wavelet (0.8)



Wavelet (0.5)

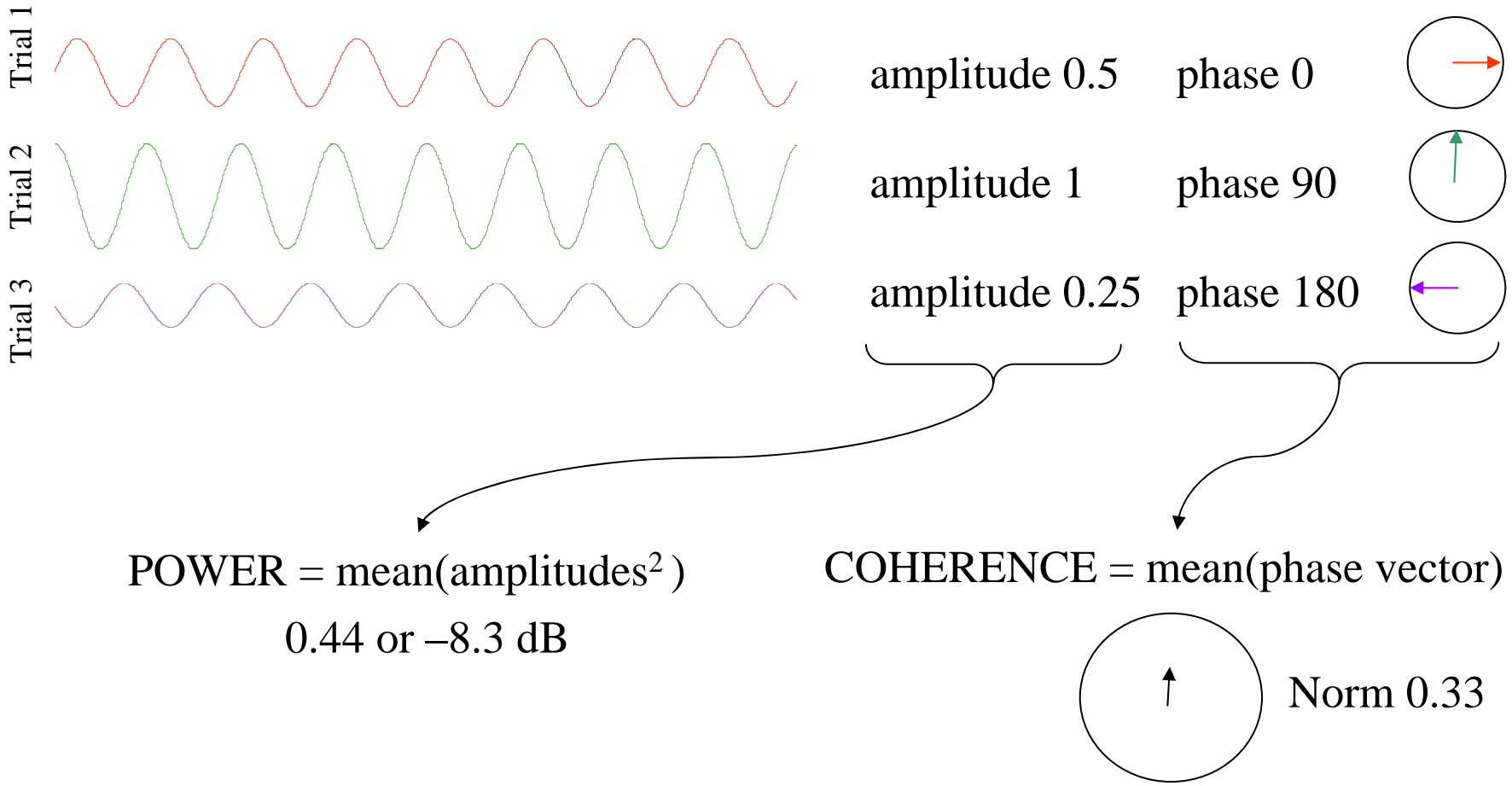


Wavelet (0.2)



# Inter trial coherence

same time, different trials



## Phase ITC

$$ITPC(f, t) = \frac{1}{n} \sum_{k=1}^n \frac{F_k(f, t)}{\underbrace{|F_k(f, t)|}_{\text{Normalized (no amplitude information)}}}$$

Normalized  
(no amplitude information)

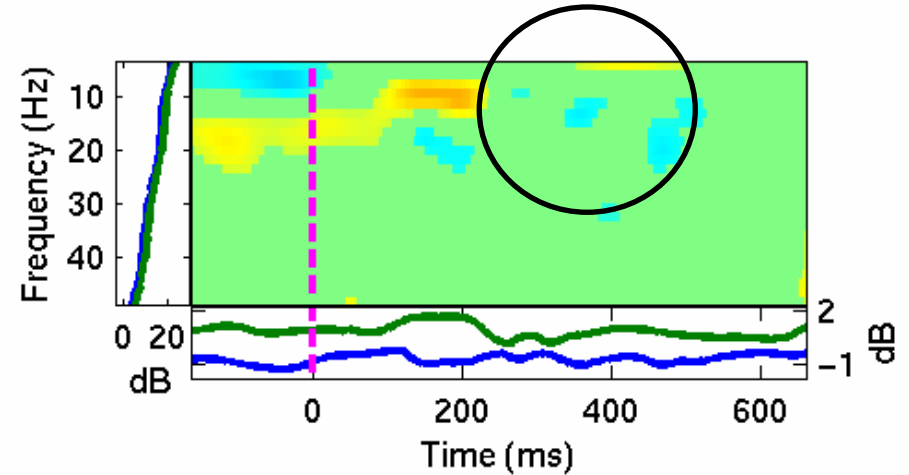
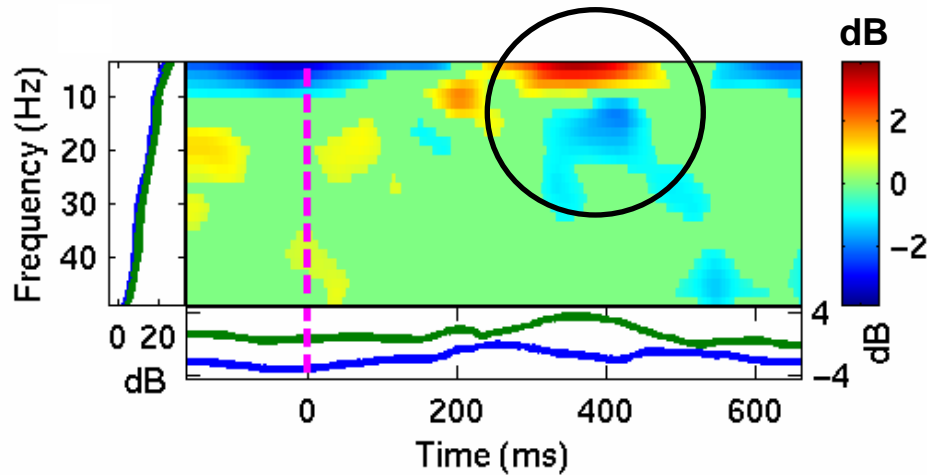
# Power and inter trial coherence

5

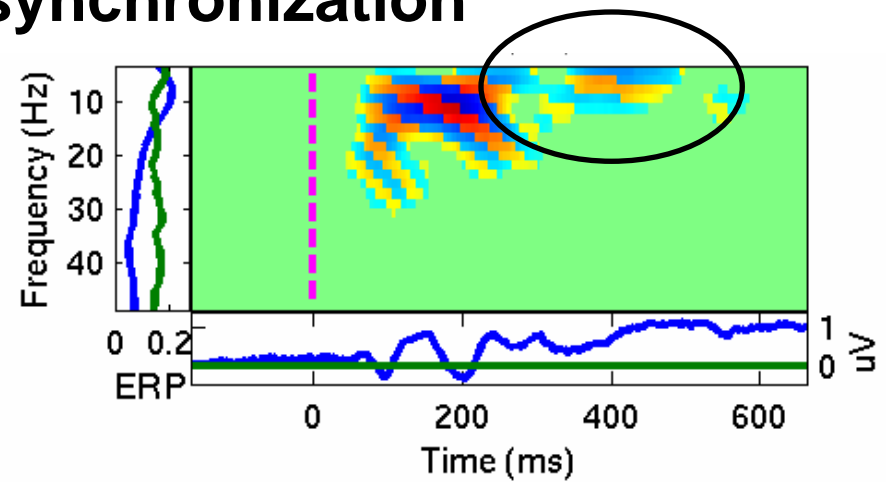
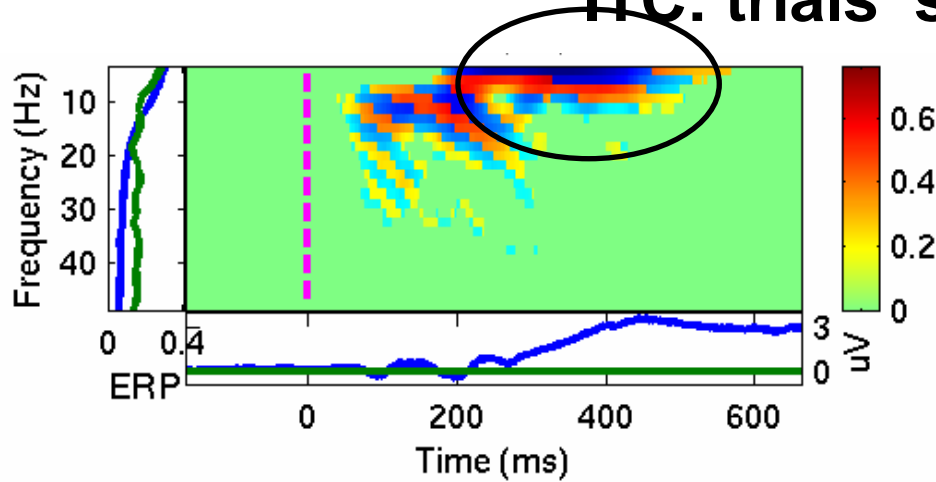
## Time-frequency power

*Condition 1*

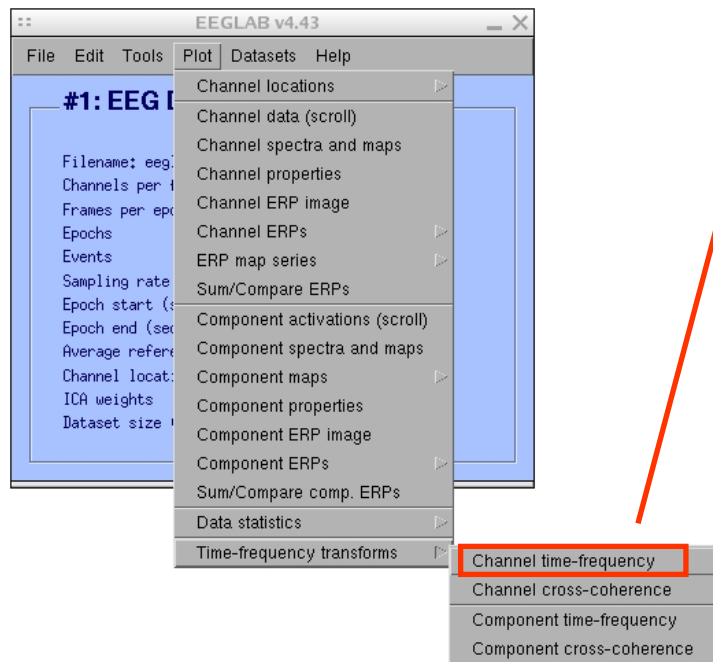
*Condition 2*



## ITC: trials' synchronization



# Channel time-frequency

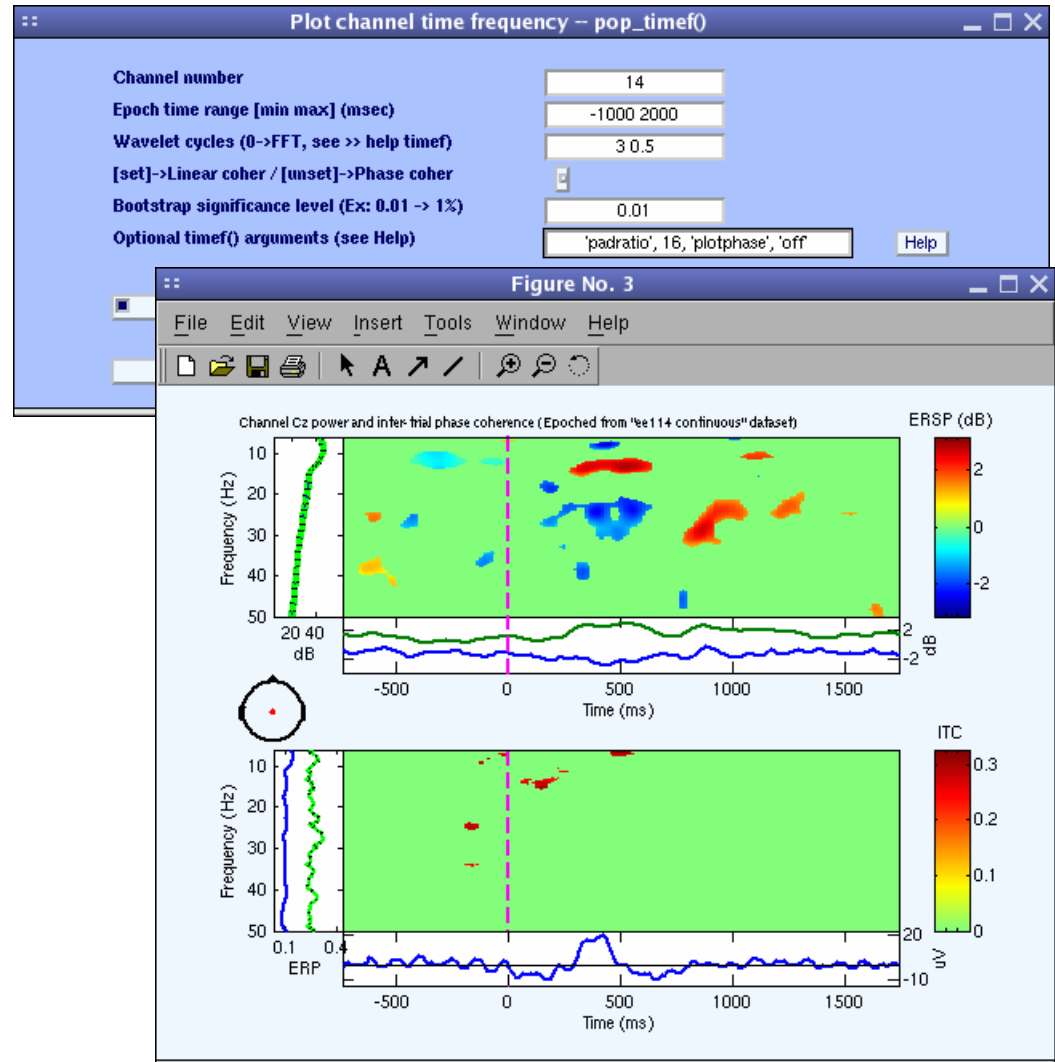


EEGLAB v4.43

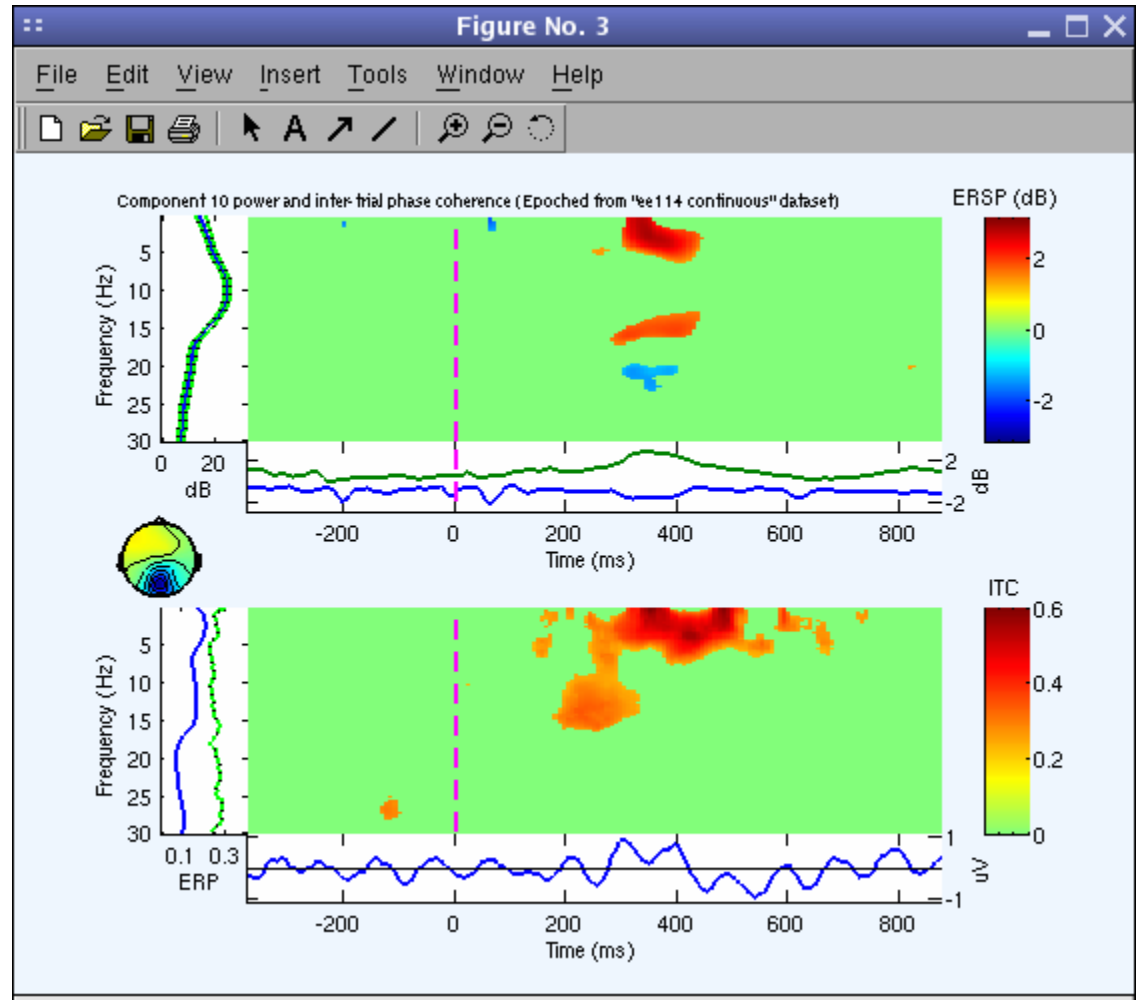
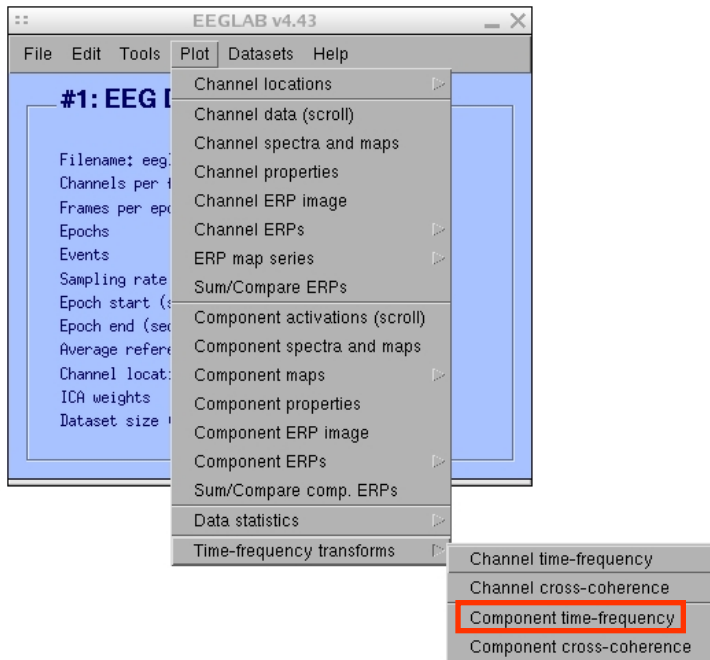
- File
- Edit
- Tools
- Plot
- Datasets
- Help

#1: EEG I

- Channel locations
- Channel data (scroll)
- Channel spectra and maps
- Channel properties
- Channel ERP image
- Channel ERPs
- ERP map series
- Sum/Compare ERPs
- Component activations (scroll)
- Component spectra and maps
- Component maps
- Component properties
- Component ERP image
- Component ERPs
- Sum/Compare comp. ERPs
- Data statistics
- Time-frequency transforms
  - Channel time-frequency**
  - Channel cross-coherence
  - Component time-frequency
  - Component cross-coherence

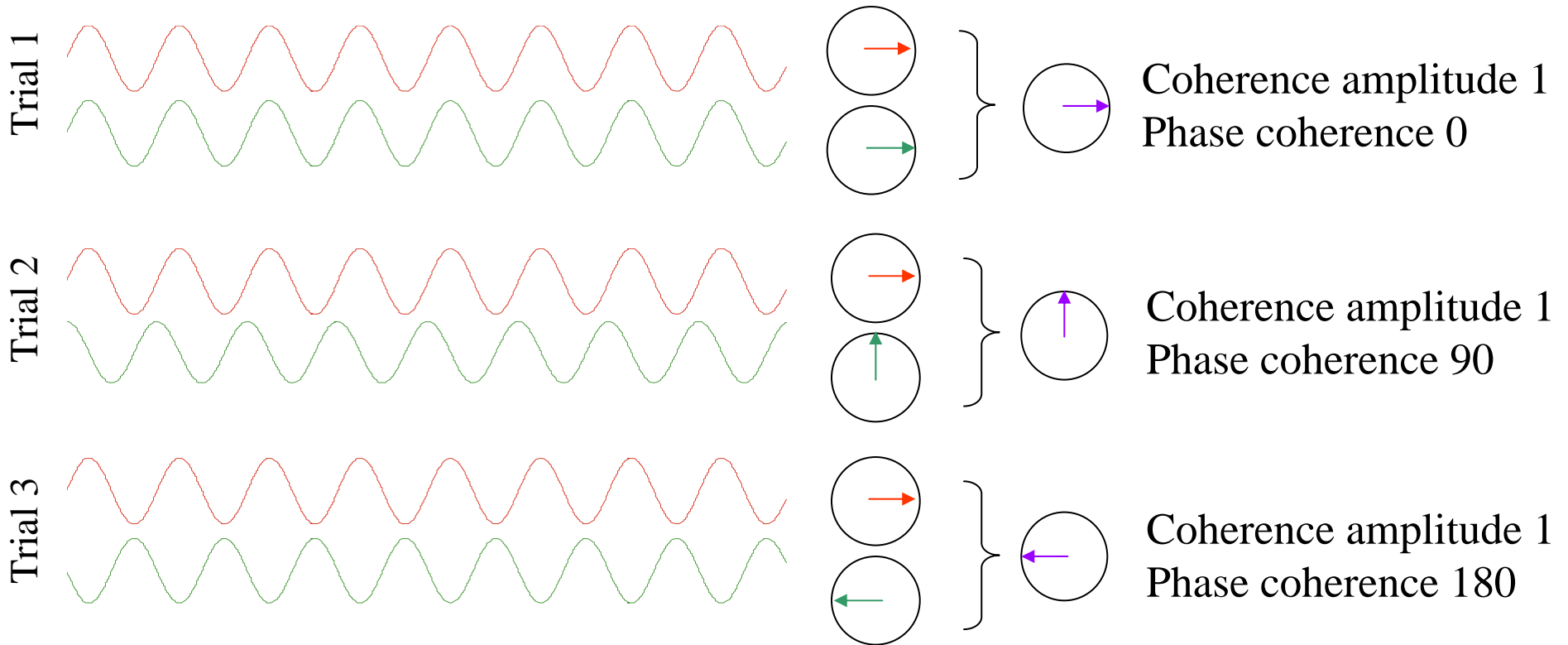


# Component time-frequency

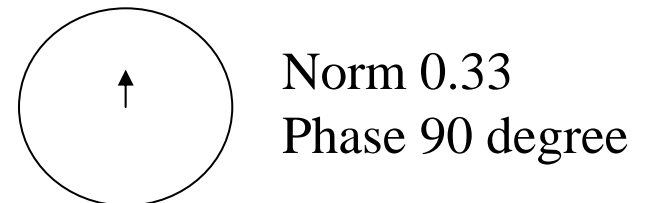


# Cross-coherence amplitude and phase

2 components, comparison on the same trials



COHERENCE = mean(phase vector)



# Phase coherence (default)

$$ERPCOH^{a,b}(f,t) = \frac{1}{n} \sum_{k=1}^n \frac{F_k^a(f,t) F_k^b(f,t)^*}{|F_k^a(f,t)| |F_k^b(f,t)|}$$

Only phase information component a

Only phase information component b

# Other spectral measures

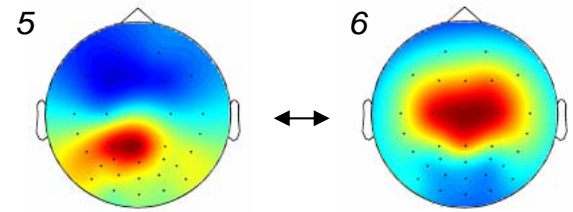
## Amplitude correlation

$$\text{corr}^{a,b}(f,t) = \frac{\text{Covariance} \sum_{k=1}^n \left( \left| F_k^a(f,t) \right| - \overline{\left| F^a(f,t) \right|} \right) \left( \left| F_k^b(f,t) \right| - \overline{\left| F^b(f,t) \right|} \right)}{n \sqrt{\frac{1}{n} \sum_{k=1}^n \left( \left| F_k^a(f,t) \right| - \overline{\left| F^a(f,t) \right|} \right)^2} \sqrt{\frac{1}{n} \sum_{k=1}^n \left( \left| F_k^b(f,t) \right| - \overline{\left| F^b(f,t) \right|} \right)^2}}$$

Standard deviation for a

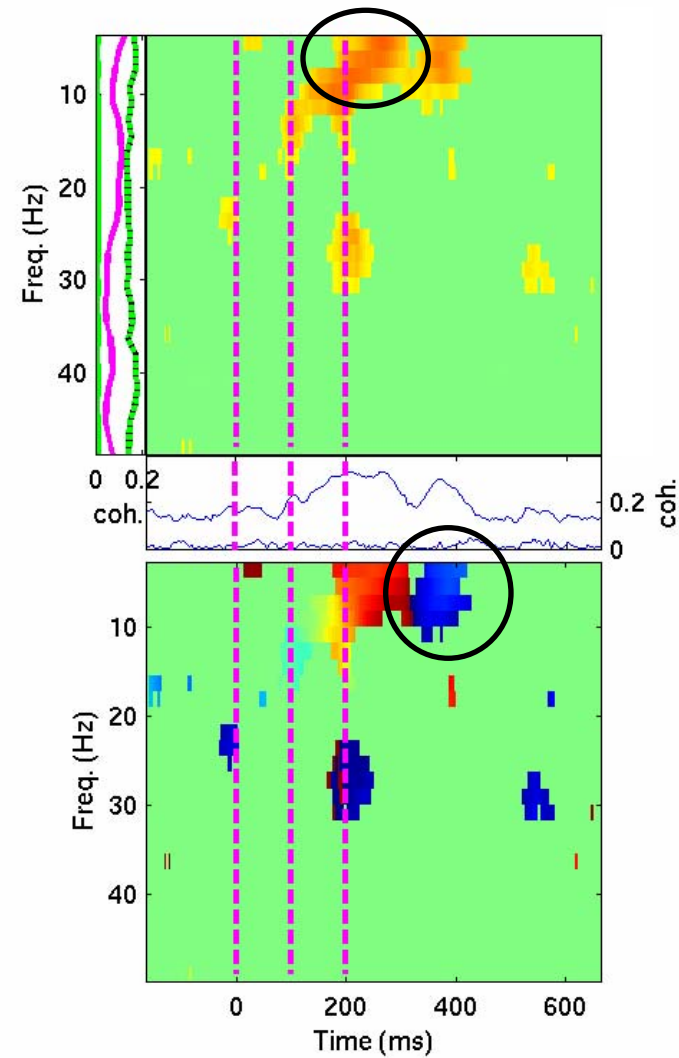
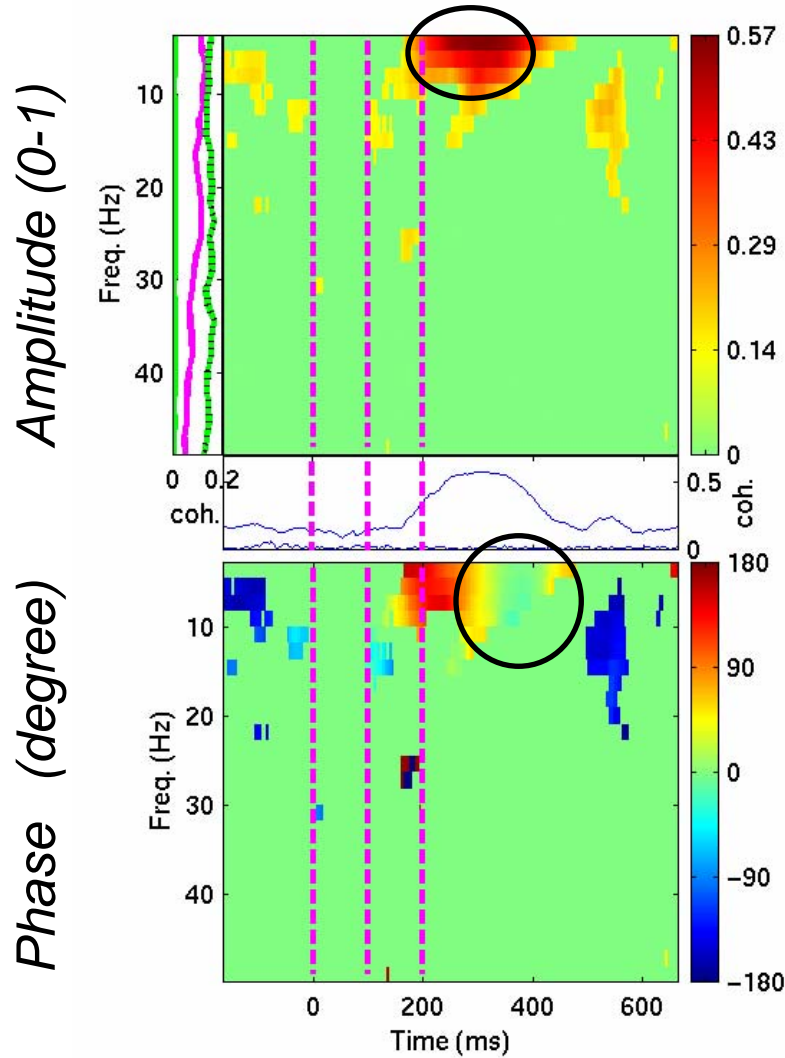
Standard deviation for b

# Cross-coherence amplitude and phase

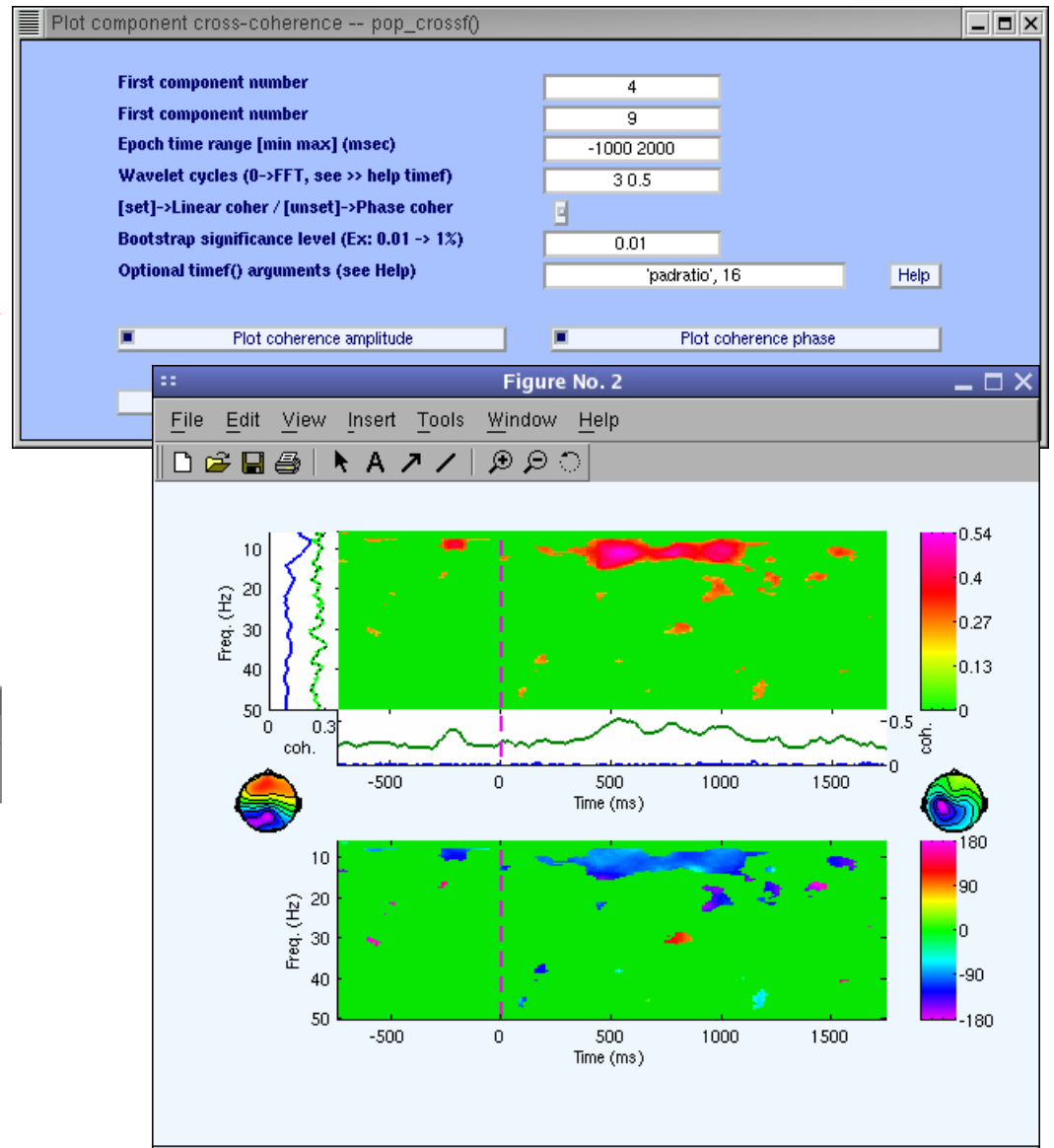
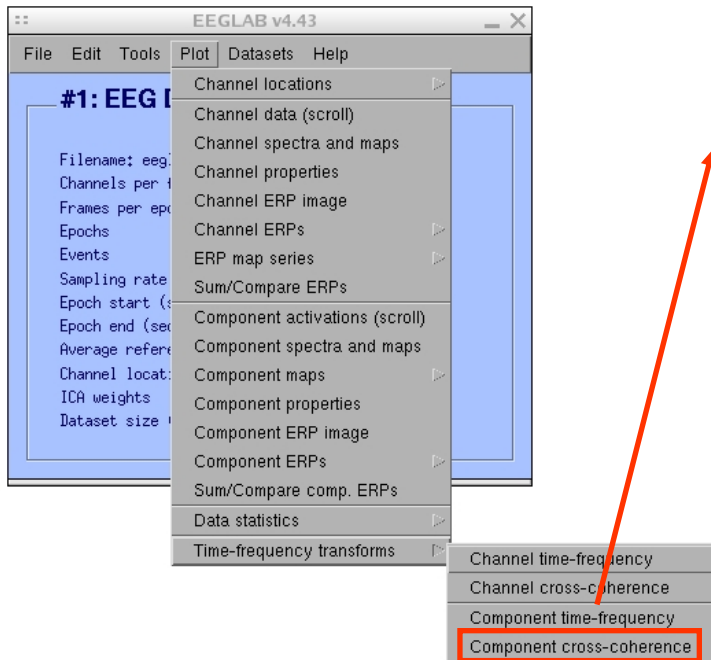


*Condition 1*

*Condition 2*



# Component phase coherence



# The Uncertainty Principle

A signal cannot be localized arbitrarily well both in time/position and in frequency/momentum.

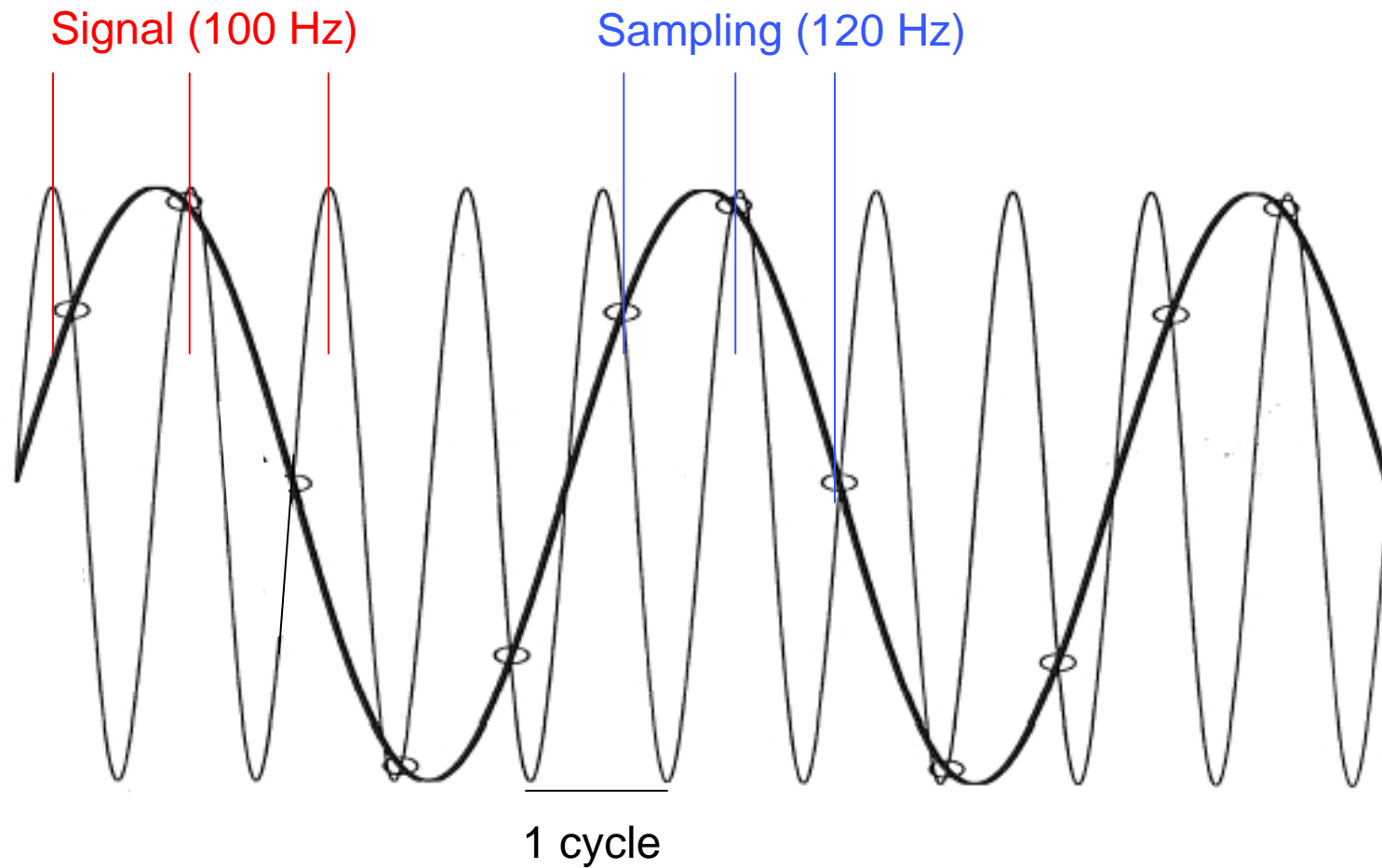
There exists a lower bound to the Heisenberg's product:

$$\Delta t \Delta f \geq 1/(4\pi)$$



$$\Delta f = 1\text{Hz}, \Delta t = 80\text{ msec or } \Delta f = 2\text{Hz}, \Delta t = 40\text{ msec}$$

# Niquist frequency: Aliasing

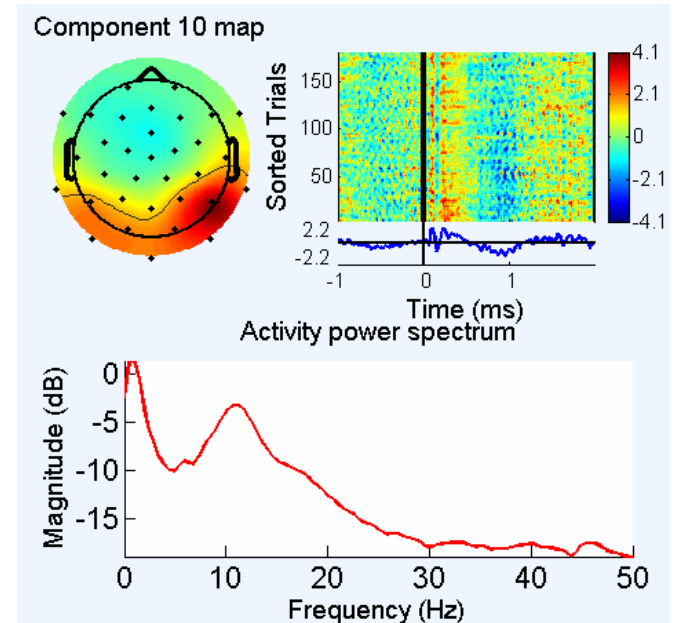
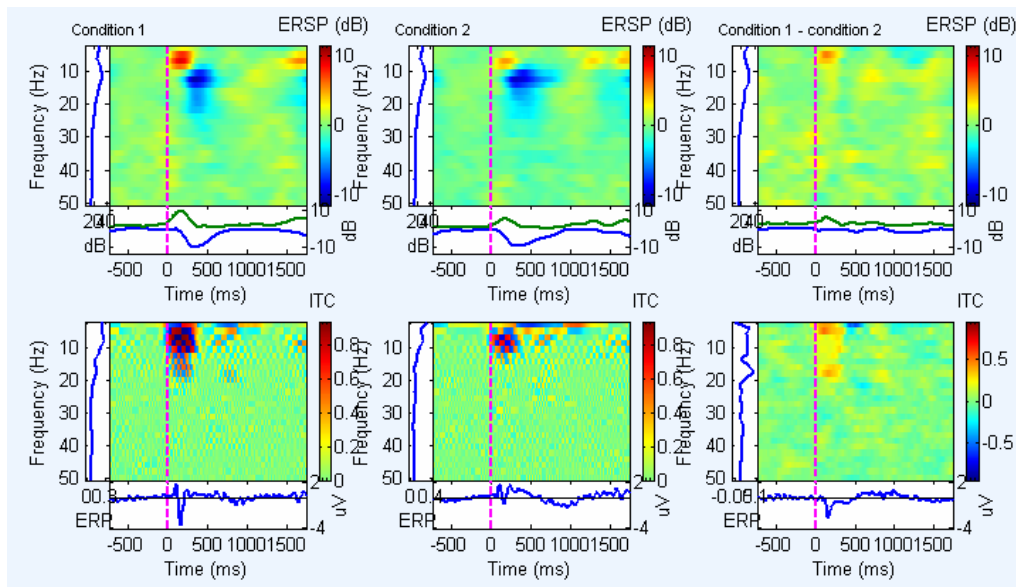


e.g. 100 Hz sampled at 120 Hz

# compare between conditions

Dataset "Face3.set"

- Extract epochs on objects
- Extract epochs on faces
- run time-frequency decomposition to compare between conditions



Component 10 for objects (left) and faces (right)

```
>> newtimef({ ALLEEG(2).icaact(10,:) ALLEEG(3).icaact(10,:) }, EEG.pnts, ...
           [EEG.xmin EEG.xmax]*1000, ALLEEG(2).srate, 0, 'padratio', 1);
```

Number of data points

Sampling rate

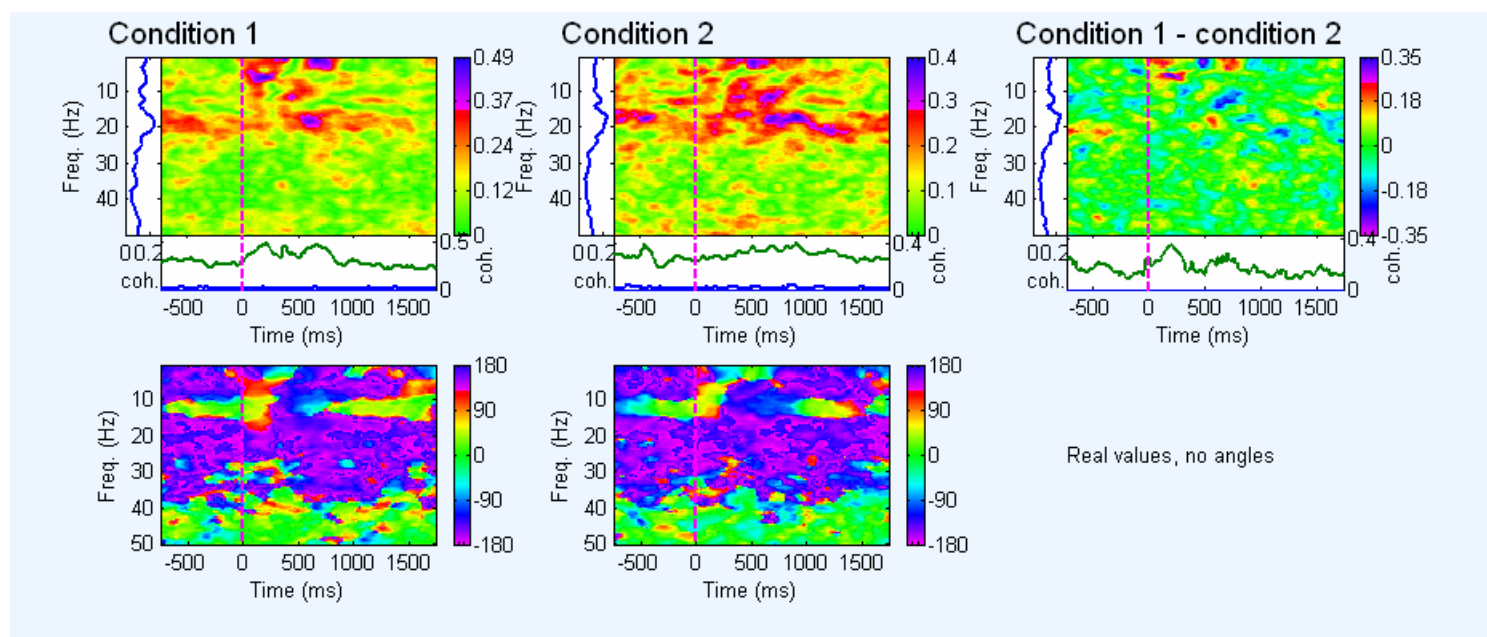
Cycles (0=FFT)

padding

Number of data points

# Compute coherence difference between components 4 and 9

```
>> newcrossf({ ALLEEG(2).icaact(10,:) ALLEEG(3).icaact(10,:) }, ...  
             { ALLEEG(2).icaact(4,:) ALLEEG(3).icaact(4,:) }, EEG.pnts, ...  
             [EEG.xmin EEG.xmax]*1000, ALLEEG(2).srate, 0, 'padratio', 4);
```



# Statistics

**Parametric statistics:** use mean and standard deviation (t-test, ANOVA, ...)

**Non parametric statistics:** do not depend on the shape of the distribution.

**Surrogate data:** shuffle data and recompute measure of interest. Use the tail of the distribution to assess significance. Semi-parametric statistics.

# Parametric statistics

Assume gaussian distribution of data

**T-test:** Compare paired/unpaired Samples for continuous data. In EEGLAB, used for grand-average ERPs.

**Paired**

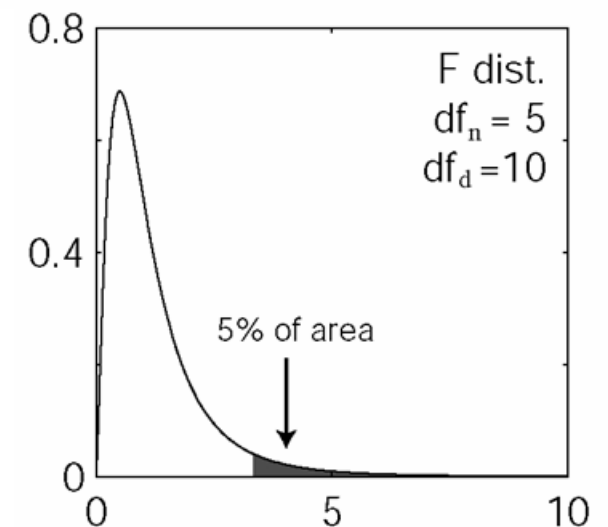
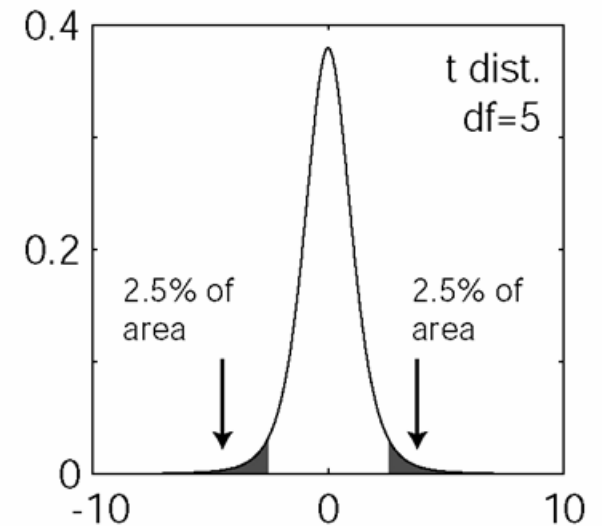
$$t = \frac{\text{Mean\_difference}}{\text{Standard\_deviation}} \sqrt{N-1}$$

**Unpaired**

$$t = \sqrt{N} \frac{\text{Mean}_A - \text{Mean}_B}{\sqrt{(\text{SD}_A)^2 + (\text{SD}_B)^2}}$$

**ANOVA:** compare several groups (can test interaction between two factors for the repeated measure ANOVA)

$$F = \frac{\text{Variance}_{\text{interGroup}} / N_{\text{Group}} - 1}{\text{Variance}_{\text{WithinGroup}} / N - N_{\text{Group}}}$$

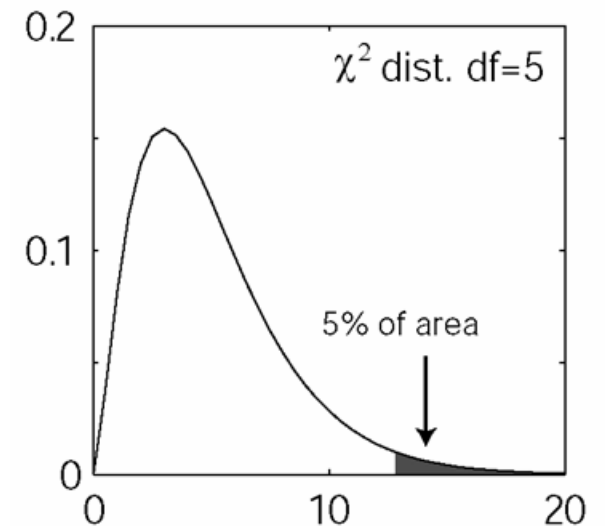
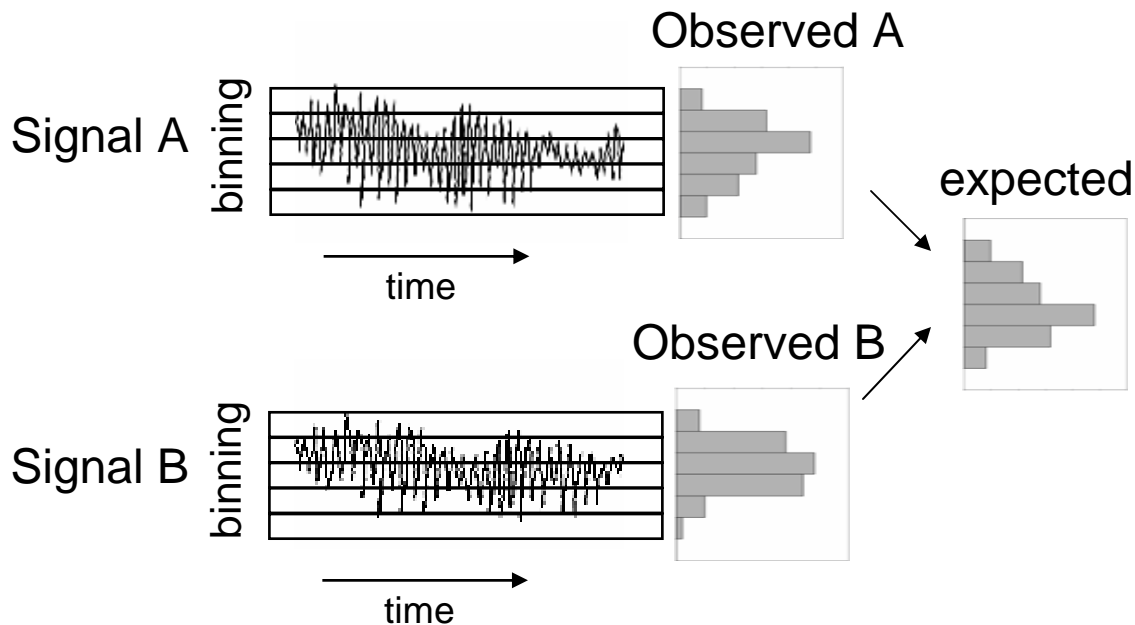


# Non-parametric statistics

Do not assume a distribution for the data

$\chi^2$  is used to compare 2 or more unpaired samples

$$\chi^2 = \sum_{i,j} (\text{Observed}_{i,j} - \text{expected}_{i,j})^2 / \text{expected}_{i,j}$$



# Randomization approach

Original data

a a a  
a a a  
a



b b b  
b b b  
b



$X_{org}$

# Randomization approach

Bootstrap or  
permutation 1

a a b  
b a a b



a b a  
b a b



$X_1$

# Randomization approach

Bootstrap or  
permutation 2

b b  
a a a  
b a

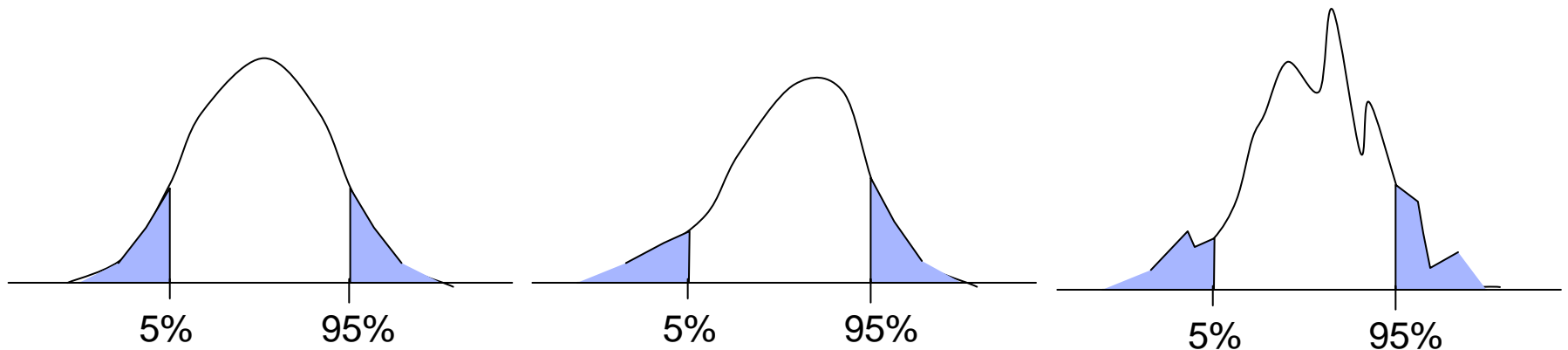


a b a  
b a b

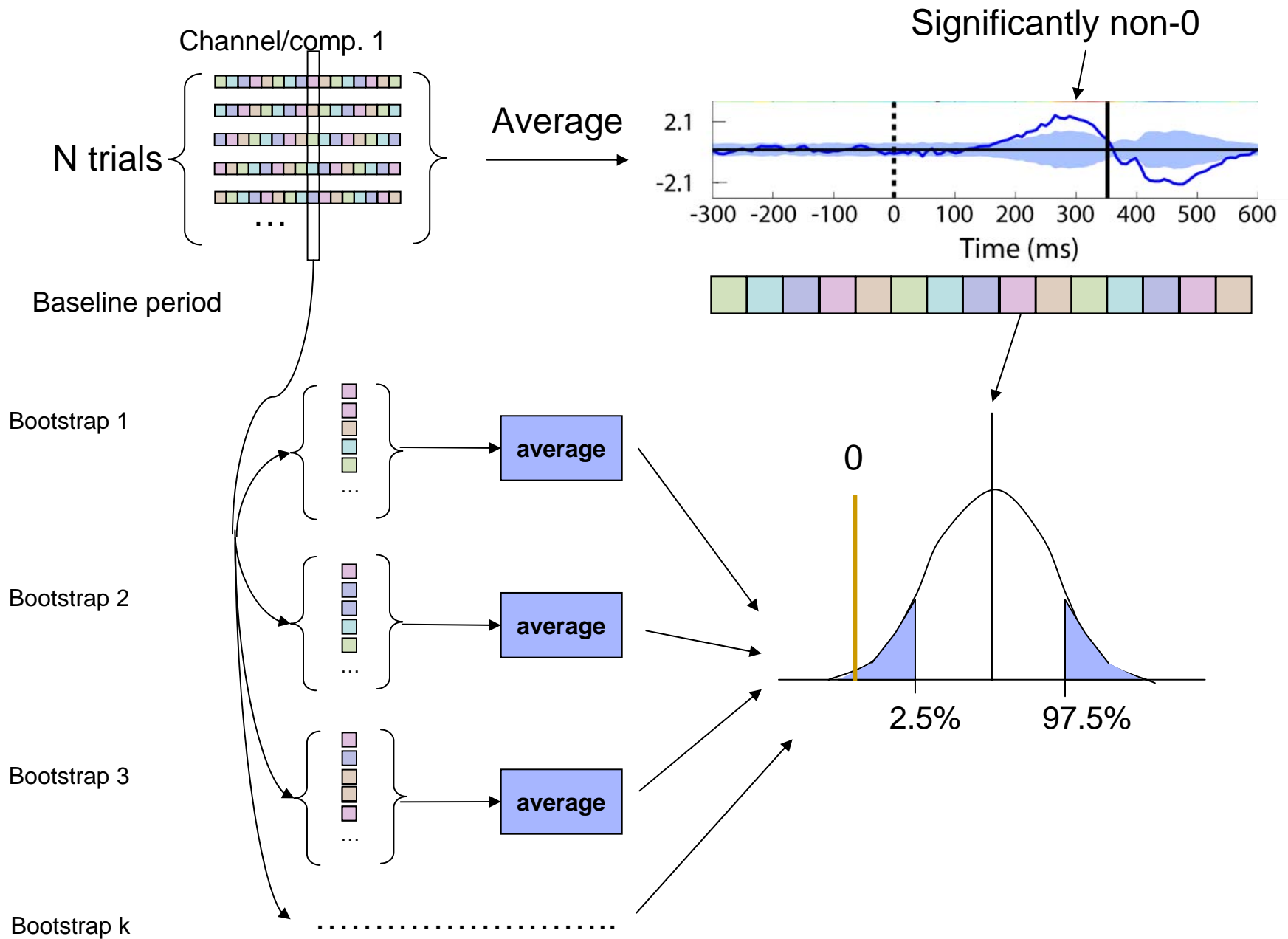


$X_2$

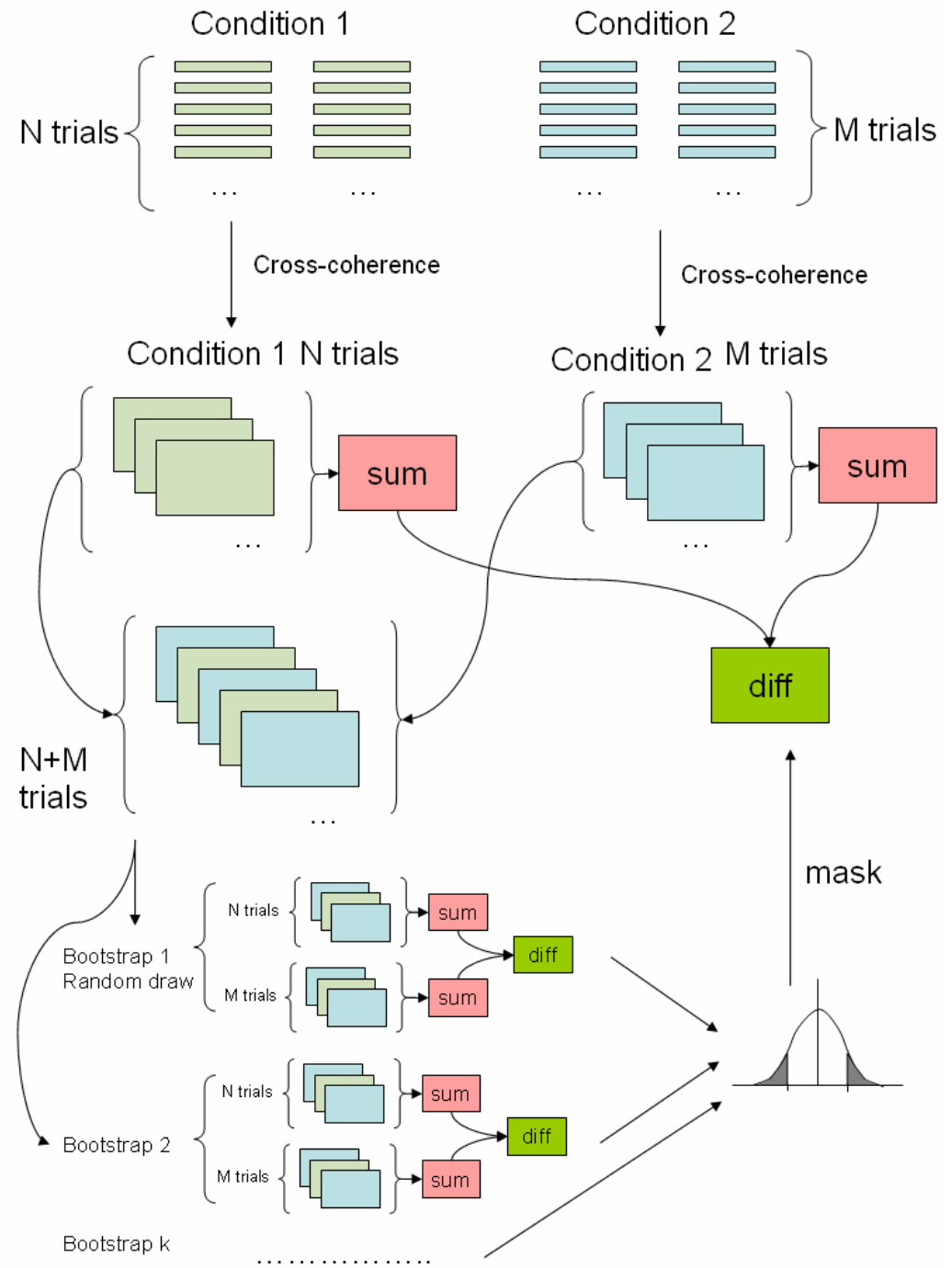
# Distribution can take any shape



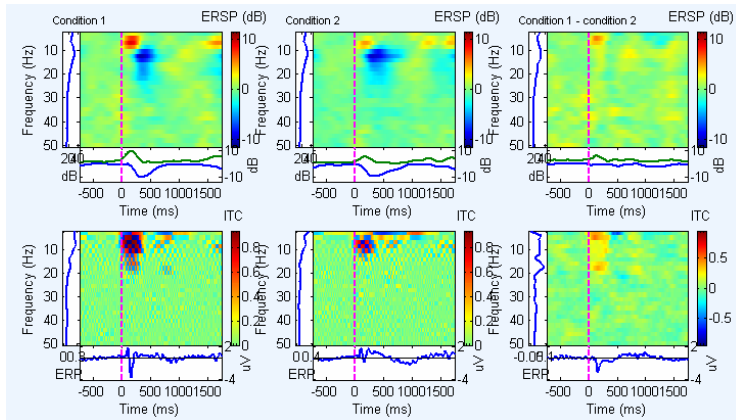
# Bootstrap for ERPs and time-frequency



# Permutation for comparing between conditions



Help mask cross-coherence difference



# Correcting for multiple comparisons

- Bonferoni correction: divide by the number of comparisons (Bonferroni CE. Sulle medie multiple di potenze. Bollettino dell'Unione Matematica Italiana, 5 third series, 1950; 267-70.)
- Holms correction: sort all p values. Test the first one against  $\alpha/N$ , the second one against  $\alpha/(N-1)$
- Clusters correction in Fieldtrip software
- MAX procedure